



with a calculator

At a time when our secondary school colleagues are embracing graphics calculators
BARRY KISSANE
suggests that it is time middle to upper primary students are given the opportunity to move on to something more than just a simple four function calculator. Why not consider a more function calculator?

Surely there can be few emerging issues in recent times for which informed Australian opinion has been more consistent than the use of calculators in schools. Similar advice is offered, whether one looks to our own professional association:

It is recommended that ALL students have ready access to appropriate technology as a means both to support and extend their mathematical learning experiences. (AAMT 1996, p.1)

[in middle and upper primary] Calculators as personal tools must be available at all times. (p.3)

or to our authoritative source of guidance on curriculum development:

All students should leave school knowing how to use a calculator effectively. It should be taken for granted that a calculator is available whenever it can be used, from Years 1-12. (Australian Education Council 1991, p.109)

or to our research evidence:

Despite fears expressed by some parents, there was no evidence that children became reliant on calculators at the expense of other forms of computation. Extensive written testing and interviews showed that children performed better overall on a wide range of items, with no detrimental effects observed. (Groves 1997, p. 157)

Despite this high level of consensus, it still seems that very many Australian children in primary schools do not enjoy unrestrained access to a calculator at school, with generally undesirable consequences. There is an assortment of reasons for this phenomenon, many of which are well documented by Swan & Sparrow (1997). As a nation, we need to do better than this. This paper assumes that we have already done so at the junior primary levels.

It seems at least plausible that, as children grow up, and pass from junior primary to upper primary and then to the secondary school, the calculator to which they have access needs to develop in sophistication as well. By the time that most of today's primary students reach the middle of the secondary school, they will need to make intelligent use of a graphics calculator, which will shortly replace the scientific calculator as a standard item of mathematical equipment for all. At the same time, the mathematical sophistication of students in upper primary school is considerably beyond that of students entering school, and it seems odd to think that the same calculator would serve their mathematical needs over the entire primary experience.

The present paper then describes briefly a calculator that might fit between these two extremes, and thus the focus is on the middle and upper primary years.

as children grow up ...
the calculator to which
they have access needs
to develop in
sophistication as well

The four function calculator

Before describing the new, it may be appropriate to make some observations about the old. Firstly, it is remarkably difficult to find examples of calculators that have only four functions, so the common descriptor is usually false. Most simple calculators have some memory functions as well as arithmetic functions; many also have a percentage key and many have a square root key.

Secondly, a surprisingly large proportion of people do not seem to know how these 'other' calculator functions work. For example, there are a number of ways percentage keys on calculators operate. To illustrate, on some calculators, the key sequence $8 \square - \square 9 \square \% \square$ will show the result (\$7.28) of applying a discount of 9% to something priced at \$8; on other calculators, the same key sequence will show only the discount itself (\$0.72), and the $\square = \square$ key must be pressed to obtain the result; on other calculators, strange things might happen: I have a calculator which gives a result of -11.1111111 for this key sequence, for example. Although it would be more convenient if all calculators worked the same way, the fact is that they don't. My impression is that many people, children and adults alike, ignore calculator operations they are not sure about (and most have long ago lost the instructions that came with the calculator.) Similar observations might be made about calculator memory operations. The task for teachers is complicated by the reality that not all students may use the same calculator, especially if home use is also considered.

Finally, it seems that little heed has been taken of the continuation of the quote from the *National Statement* above:

A number of skills, both conceptual and technical, underlie the correct use of a calculator and these should be taught explicitly. (Australian Education Council 1991, p. 109)

Relatively few children to whom I have spoken report being explicitly instructed on when, how and why to use their four function calculator, although many report instances of being told that they were not permitted to use their calculator for a particular purpose. To develop sound calculator use in children, we need not only to know how calculators work but also to devote some explicit attention to helping children know how their particular calculators work, as a precursor to thinking about how and when to use them. It is not necessary (and arguably not even desirable) to have special 'calculator activities'

for these sorts of educational purposes. Clearly, preventing children from using a calculator will not help them make decisions for themselves about which form of technology to use. Such a conceptual discrimination skill must be taught explicitly, and will not be learned by osmosis, as the *National Statement* recognised.

The more function calculator

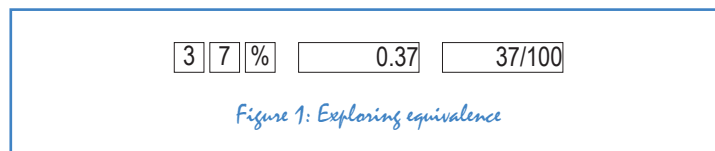
Until recently, the next step from a simple calculator was a scientific calculator, the most obvious extra features of which are mathematical tables (such as those for trigonometry, logarithms and exponential functions) and numerical data analysis (such as calculation of means and standard deviations). These mathematical features are not necessary for the mathematics of the primary school. However, there are now less drastic, and much more useful extensions of the simple calculator available, and these are the subject of the rest of this article.

The most useful extra features are those concerned with fractions. As children progress through middle and upper primary school, the extension of the numbers with which they deal from integers to fractions and decimals becomes important. All calculators

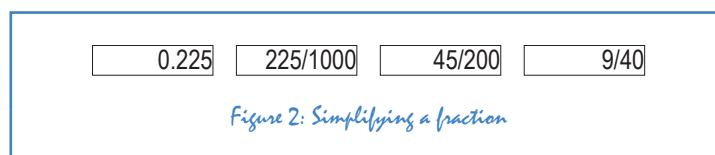
routinely deal with decimals, but there now exist calculators for primary school children that deal just as routinely with fractions and also with the relationships between fractions and decimals.



when necessary for mixed numbers. The $\boxed{\text{F-D}}$ key toggles a result between fractions and decimals. Entering $9/10$ and pressing the $\boxed{\text{F-D}}$ key a few times switches the display between $9/10$ and 0.9 . Entering 37% on the calculator displays 0.37 , and pressing the $\boxed{\text{F-D}}$ key displays $37/100$, thus helping students develop a feel for the relationships between different representations of numbers, as represented in Figure 1.



By their nature, terminating decimals have fractional representations with powers of ten in the denominator, but frequently we would prefer simplified equivalent versions. The calculator handles this by having a simplification key $\boxed{\text{Simp}}$, which automatically reduces a fraction by the lowest integer possible. (The display has a symbol that indicates when simplification is possible, but the decision to simplify is at the discretion of the person pushing the keys.) For example, pressing this key after converting the decimal 0.225 to a fraction (shown as $225/1000$) results in the sequence of displays shown in Figure 2.



There are a number of calculator models with such capabilities. However, most of them are presently too expensive for primary school children to be expected to own. Examples in this category include the Sharp EL-E300 and the Casio fx-55. So, to illustrate these extra capabilities, we use Texas Instruments product, the *Math Explorer* calculator, which is more modestly priced at around \$5 or \$6 per year, assuming that students use it for about four or five years. Personal ownership is a necessary, but not a sufficient, condition for unrestricted access to a calculator; occasional, or even regular, access to a class set of calculators does not provide the same level of assurance.

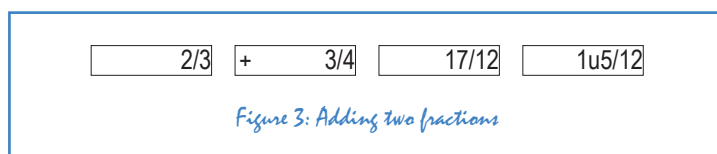
Fractions are readily entered on the *Math Explorer* using a fraction separator key $\boxed{/}$, and a $\boxed{\text{Unit}}$ key

It is possible to find out which factor has been used automatically by the calculator to simplify a fraction, and it is also possible to specify which factor to use, thus making the *Math Explorer* calculator a wonderfully versatile environment in which children might explore the connections between fractions and decimals (and percentages), with some prospect of strengthening their conceptual grasp of each. Of course, the calculator also allows for standard operations, so that as well as converting and simplifying as shown in Figure 2, the operation $\boxed{9} \boxed{\div} \boxed{40} \boxed{=}$ will yield 0.225 , offering opportunities to explore the crucial connection involving division.

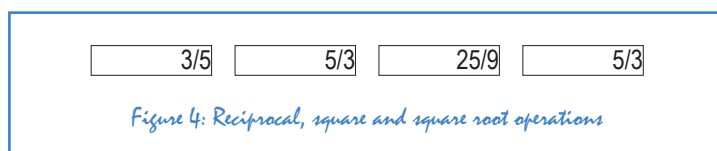
There are limitations, of course, since the screen displays only eight digits. So, the decimal to fraction conversion works only up to three decimal places, and even less if the decimal contains a whole number part. (Sensibly, the calculator does not report an error; it simply doesn't respond to the request to convert.) These are not severe problems, and may even be used to advantage in understanding the decimal representation of fractions. For example, the calculator will obligingly convert $2/3$ to 0.6666667 , but will not then convert back again, which can be used to good educational advantage.

As well as representing numbers, calculators are useful for manipulating them. On the *Math Explorer*, addition, subtraction, multiplication and division of fractions are as routine as decimal computations. Results are given as fractions, which can be simplified at the discretion of the child, or converted to decimals if desired. An example of successive displays for the addition of $\frac{2}{3}$ and $\frac{3}{4}$ is shown in Figure 3.

Figure 3 also shows (in the final display) that improper fractions can be converted to mixed fractions if desired (using another key labelled $\frac{Ab}{c}$). Fractions can also be entered into the calculator in this way, with the 'u' representing units.



Like the other calculators mentioned above, the *Math Explorer* also has some other operation keys which also work with fractions. There is a reciprocal key $\frac{1}{x}$, which will turn a fraction upside down, a squaring key x^2 , which will square a fraction and a square root key $\sqrt{\quad}$, which will give a fractional square root when possible, or a decimal when necessary. Figure 4 shows the results of using these three keys in succession after entering the fraction $\frac{3}{5}$.



Armed with such a device, students do not need to rely on complicated and ill-understood algorithms for manipulating numerators and denominators separately in order to complete elementary arithmetic with fractions. Of course, in many cases, as for operations with other kinds of numbers, informal and mental methods are preferred over paper and pencil and calculator methods. An experienced and older child using a fraction calculator to find $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$ rather than thinking of it as $\frac{5}{10} + \frac{2}{10}$ would be just as much a source of concern as one who used the standard paper and pencil algorithm for the same purpose. What the calculator provides, which paper and pencil algorithms do not, is the assurance that any likely fraction calculation can be completed correctly, as well as ready access to some of the connections among fractions and between fractions and decimals. For children growing through the primary years, it seems very likely that access to a device of this sort will help them learn the most important things about fractions.

It may even be the case that we may tackle some mathematical tasks differently with ready access to a fraction calculator. For instance, instead of dealing with time in hours and minutes, we may deal with it as hours and fractions of an hour: 3 hours and 23 minutes can be represented as 3 and $\frac{23}{60}$ hours on the calculator, and operations can be performed more easily than they can with base 60 arithmetic.

An experienced and older child using a fraction calculator to find

$$\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$$

rather than thinking of it as

$$\frac{5}{10} + \frac{2}{10}$$

would be just as much a source of concern as one who used the standard paper and pencil algorithm for the same purpose.

The death of long division?

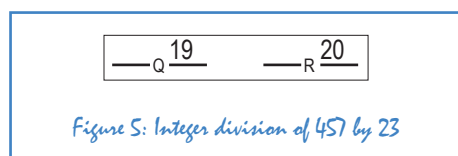
The school mathematics curriculum has more than its fair share of sacred cows, items that seem to persist regardless of common sense. We might take heart from the disappearance of some sacred cows, such as extracting square roots of numbers using a paper and pencil algorithm, which is (thankfully) rarely seen these days. Long division seems to be a good example in the primary school, and has already proven itself to be quite resistant to attacks by people wielding calculators.

While some of the arguments for doing things the hard way when there are easier alternatives appear to be nothing more than nostalgic, there have also been suggestions that students will 'not really understand

what they are doing' unless they use the paper and pencil algorithm for long division; that such a suggestion is plainly ridiculous is easily established by talking to the vast number of adults who can competently use the algorithm without a trace of understanding of what is going on. But in the particular case of long division, the results provided by the obvious alternative technology of calculators have indeed been a little troublesome, when it is desired to know the remainder after dividing one integer by another. Although almost never (except in school) do we actually need to know the remainder in any practical situation, it is nonetheless true that it is hard to find out what it is when using a typical simple calculator.

To give an example, on a standard calculator, $457 \div 23$ gives a result of 19.869565. To find the remainder, one must either recognise the decimal part, 0.869565, as a particular fraction with denominator 23 (an unlikely event for most adults, as well as children) or convert it into one (by first subtracting 19 from 19.869565 and then multiplying the result by 23), which is a bit slow and messy. An alternative is to compute $457 - 19 \times 23$, after inferring from the calculator display that 23 must go 19 times into 457, a process which also takes more time than we might like to spend regularly.

The new breed of calculators, like the *Math Explorer*, perform integer division with a single command, using a key labelled $\boxed{\text{INT}\div}$. The result of using this command for the example of 457 divided by 23 is shown in Figure 5.



Clearly, interpreting the calculator display in Figure 5 demands that children understand that 23 goes into 457 nineteen times with twenty left over; the 'Q' in the display stands for

quotient and the 'R' for remainder. With ready access to such a command, children can be expected to make a decision on whether or not a result ought be given exactly or approximately, and if exactly, which of a decimal or fractional answer makes more sense. Perhaps, too, widespread access to such technology might finally sound the death knell of long division. And not before time.

Conclusion

There is not space here to describe in detail ways in which calculators like Texas Instruments' *Math Explorer* might be used by children and their teachers to good effect. As was suggested above, unrestricted access by itself may well be the best way to generate productive uses.

Finally, it is interesting to note that the graphics calculators that have been produced for the lower secondary school, most notably the Texas Instruments TI-80 and the Casio fx-7400G each contain similar fractional capabilities to those described above as well as integer division facilities. However, it seems to me too long to wait from the beginning of primary school, when a simple calculator is regarded as necessary for all students, until some time in the secondary school, when students might be expected to have access to such machines. Growing up mathematically with a calculator might proceed more sensibly if there were a device of the kind described here somewhere in the middle of the experience – and students were permitted to graduate from a four function calculator to a more function calculator.

References

- Australian Education Council (1991). *National statement on mathematics for Australian schools*. Carlton: Curriculum Corporation.
- Australian Association of Mathematics Teachers (1996). *Statement on the Use of Calculators and Computers for Mathematics in Australian Schools*. Adelaide: AAMT.
- Groves, S. (1997). The effect of long-term calculator use on children's understanding of number: Results from the Calculators in Primary Mathematics project. In Scott, N. & Hollingsworth, H. (Eds) *Mathematics: Creating the future*. (pp. 150–158). Melbourne: AAMT.
- Swan, P. & Sparrow, L. (1997). Impediments to the use of calculators in the primary school. In Scott, N. & Hollingsworth, H. (Eds) *Mathematics: Creating the future*. (pp. 283–288). Melbourne: AAMT.

Barry Kissane works at Murdoch University and has a keen interest in the use of technology in the classroom. He has published widely, particularly in the use of graphics calculators.