

INTERACTIVE MATHEMATICS ON THE *CLASSPAD 300**

Barry Kissane

Murdoch University, WA

This paper describes some of the fresh opportunities for learning mathematics that have become available with an interactive hand-held device, the Casio ClassPad 300. Some of the different ways in which students can be engaged interactively in mathematical work are described and illustrated. These include interactions on a touch-sensitive screen via a stylus, drag-and-drop interactions between applications, the use of two windows simultaneously, the use of measurement bars, the deployment of dynamic links between windows and the concept of an eActivity, designed for organizing student interactions for a particular purpose. It is argued that interactions of these kinds have considerable potential for learning mathematics.

Introduction

The main aim of this paper is to explore the notion of interactive learning of mathematics on hand-held devices. Two of the attractions of hand-held technologies for mathematics are that they are small enough to be portable and that, by their nature, they involve students directly in mathematical work. Another convenient by-product of being hand-held is that there are good prospects that educational authorities, such as Boards of Studies, will be able to incorporate their use sensibly into mainstream mathematics curricula, even those with external examination requirements. The most obvious disadvantages of hand-held technologies over other forms of ICT involve the software limitations, screen resolution problems and communication limitations. However, it seems inevitable that the advances of technology will address these disadvantages. Indeed, as Leigh-Lancaster, Norton & Evans (2003) noted, approximations to some of these are already available. In this paper, a recent one of these, the Casio *ClassPad 300*, a photograph of which is shown in Figure 1, is described in a little detail and its implications for interaction in particular and for teaching and learning mathematics in general is explored.

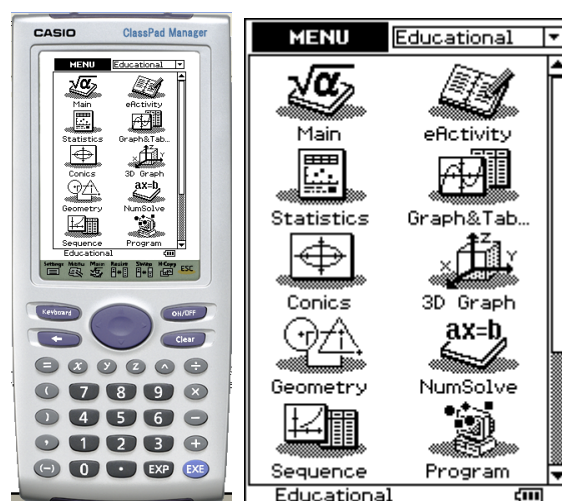


Figure 1: Photograph and opening menu of *ClassPad 300*

To date, graphics calculators have been the main manifestation of the idea of a personal technology for learning mathematics. For some time, however, the term ‘calculator’ has been recognized as problematic for describing hand-held technological devices. For example, almost a decade ago, Kissane (1995, p. 161) suggested that graphics calculators were better described as small hand-held computers than as calculators, a problem that has worsened somewhat since the inclusion of computer algebra systems (CAS). Even the adjective ‘graphics’ is problematic, since it attends to the nature of the display screen rather than to what is displayed on it – although it is a superior choice to ‘graphing’, which tends to reinforce a common misconception, arising from the unfortunately limited view that graphics calculators are principally calculators that can be used to draw graphs (Kissane 1997). The details of the screen of the *ClassPad 300* in Figure 1 might at first give the impression that the device is similar to a graphics calculator, with a larger screen and less keys; in some ways, this is a reasonable interpretation, as the device is about the same size and mass as a typical high-end graphics calculator. However, although devices like the *ClassPad 300* and Texas Instruments’ *Voyage 200* have some clear links to graphics calculators (e.g., graphics calculator capabilities are routinely included within their operations), the errors associated with the term ‘calculator’ would be seriously compounded if it were used to describe them. The remainder of this paper is concerned with illustrating how interactive activity is manifested on the device, to help make it clear how it differs from a graphics calculator.

Interaction and learning

Technology has the potential to play a number of roles in mathematics education. For example, Kissane (2002) distinguished three different roles: computational, influential and experiential, exemplifying these for the case of the graphics calculator. In the present paper, the focus is on the experiential role, as a device like a *ClassPad 300* offers new ways of experiencing mathematical ideas, experimenting with them and seeing their connections. The crucial feature that is new here involves interaction. The normal meaning of interaction when discussing mathematics teaching involves interaction between people, such as students or teachers. While this kind of human interaction is likely to occur also with classroom use of devices like this, it is not the main point in this paper. Interaction here refers to interactions between the user and the mathematics. Dreyfus (1994) observed the importance of this idea:

Computers make it possible to represent mathematics visually, by means of diagrams, with an amount of structure not offered by any other medium. Graphic computer-screen representations of mathematical objects and relationships allow for direct action on these objects (rather, their representatives) and observation of the ensuing changes in the diagrammatically represented relationships; this in turn may help the student to realize the existence and understand the nature of relationships. (p. 204)

Similarly, Kaput (1992) described the importance of interactivity in computers generally, helping to provide some theoretical support for an essentially interactive device like a *ClassPad 300*:

Interactivity of the computer medium strongly distinguishes computers both from static media and from traditional video media. If you write a statement in a static medium such as pencil and paper, it merely sits there and does not interact with either the paper or the structure of anything else that may have been written on that paper. Further, the paper does not provide much in the way of help or constraint to your writing ... the key difference with notations instantiated in interactive media is the addition of something new to the result of a user’s actions, something that the user must then respond to. (p. 526)

The *ClassPad 300* provides a number of different ways of users interacting with mathematical ideas, consistent with these earlier views of the importance of doing so. In the rest of this paper, a brief description and some illustrations of these are provided.

Some kinds of interaction

The ClassPad 300 offers several kinds of interaction, briefly described and exemplified below.

Use of stylus for moving objects

Most interactions between user and *ClassPad 300* take place through the stylus on the touch-sensitive screen, which provides a ‘hands-on’ feeling for many actions. Figure 2 shows an example of this in which a matrix inversion is shown in the first screen. In the middle screen, the algebraic expression $a + 2$ is dragged to the matrix before inversion, substituting for k . The final screen shows the results of doing this. As well as such drag-and-drop operations, a cut and paste equivalent is available, again through the medium of the stylus. This gives a feel for the idea of substitution, as the k is literally replaced by the $a + 2$.

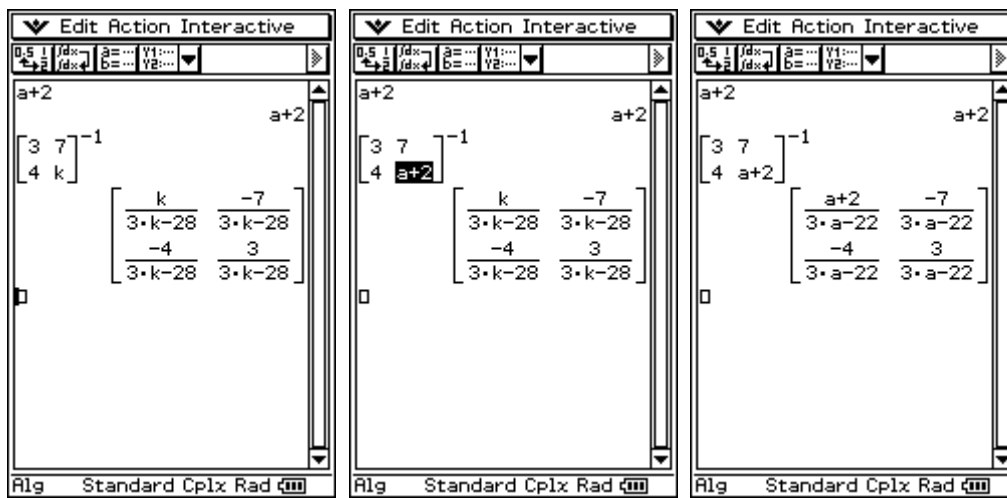


Figure 2: Dragging and dropping an expression to substitute for $k = a + 2$

As a second example, the *ClassPad 300* includes a *constraint-based geometry* capability (HREF1), which has many similarities with dynamic geometry software on computers (and calculators). Using a stylus, objects are created, moved, animated and explored in a very direct way; the feeling is akin to drawing objects with one’s finger, which feels a bit less removed from the essential operations than moving a (more distant) mouse on a computer or cursor keys on a calculator. Figure 3 shows an example of moving vertex B of a triangle ABC to verify that the medians are concurrent, irrespective of the position of the vertices.

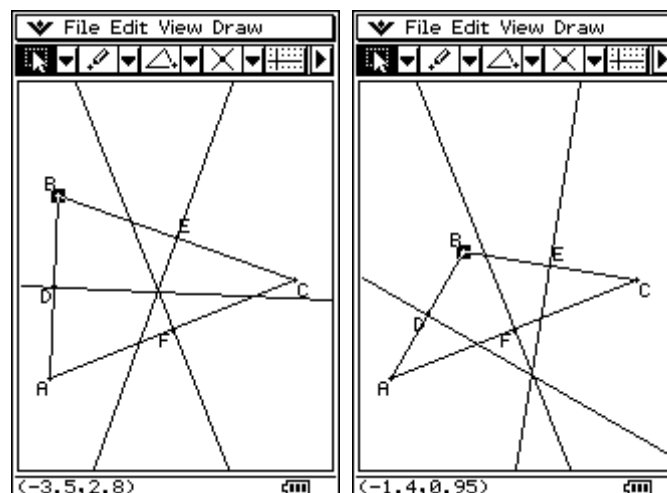


Figure 3: The medians of a triangle are concurrent

Dual windows

Another sense of interaction is that between different ways of looking at things. Much has been said of the merits of multiple perspectives for learning, and a device that allows for more than one ‘perspective’ to be visible at once seems to offer new learning opportunities for interaction. The interactions here are in the mind of the user: grappling with how the different representations of a function can be reconciled is helped by seeing two of them at once; in contrast, on most graphics calculator screens, only one representation at a time is visible. Figure 4 depicts an example of this opportunity for a cubic function.

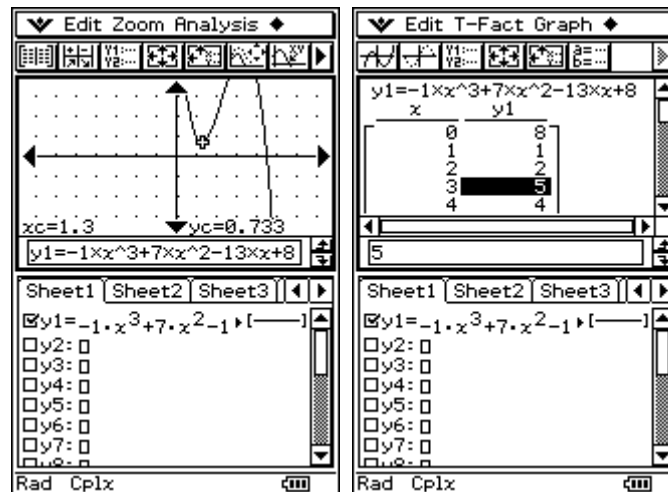


Figure 4: Interactions between symbolic, graphical and tabular representations

Choosing to interact with more than one window at once is facilitated by the ready accessibility of different windows in a drop-down menu. In the example shown in Figure 5, from the Main screen, users can select a variety of potentially useful windows to explore links; those shown in the drop-down menu in the first screen include a 2D or 3D graph window, function definition windows of different kinds, statistical windows, a geometry window and a numerical equation solving window.

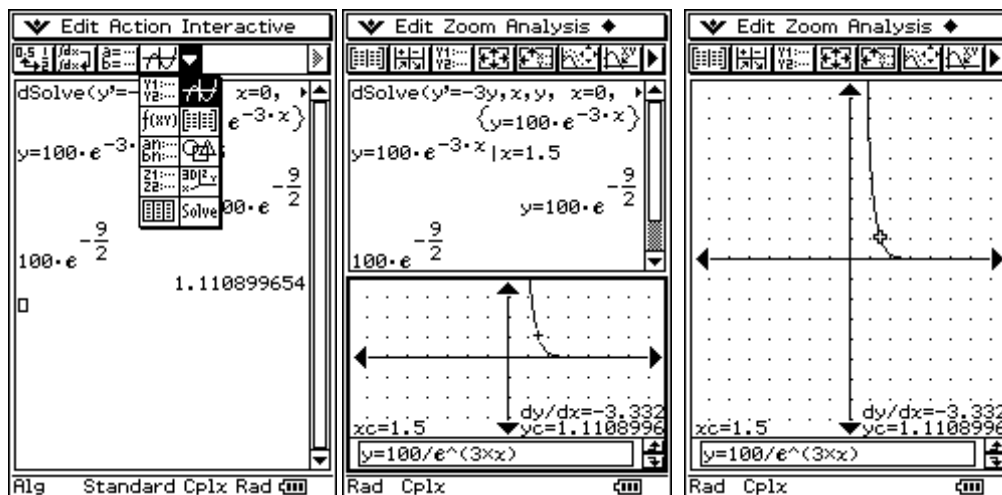


Figure 5: Choosing a second window to explore meanings associated with a differential equation

In this example, the general solution of a differential equation with initial condition (0,100) using CAS in the Main screen is explored graphically in the middle screen. Dragging and dropping the symbolic solution to the graph window helps to see the links between the numerical and graphical work. Small windows can be resized to full screen if desired, as shown in the third screen.

Movement between windows

The previous example illustrated the idea of ‘dragging and dropping’, a particularly powerful form of interaction, as it allows students to see almost instantly different versions of the same mathematical object. Figure 6 below shows how a window can be added to the bottom of the Main window, which contains a symbolic statement of the quadratic function, $y = x^2 - 1$. In this case, a Geometry window has been chosen. The effect of the dragging and dropping is to represent the function graphically.

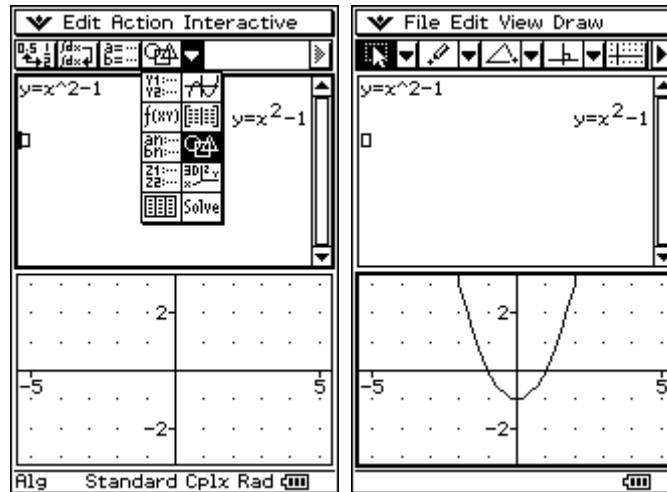


Figure 6: Dragging and dropping a function into the Geometry window

Once represented, the graph itself can be manipulated directly by the user. The first screen in Figure 7 shows that the graph has been dragged (using the stylus) to the right and down, so that it no longer represents the original function. To see what function is now represented, the final screen in Figure 7 shows the effects of dragging and dropping the new graph back to the main screen, where a symbolic representation of the function graphed is shown. In this case, the *ClassPad 300* routinely shows the transformed version of the function, which makes it clear that the graph was moved three units to the right as well as one unit (further) down.

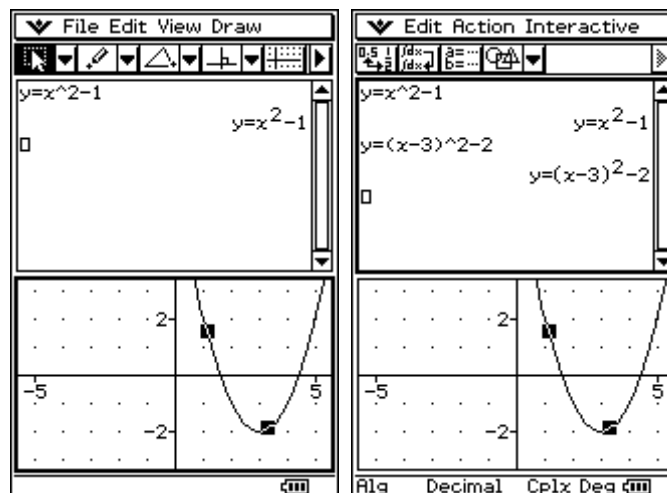


Figure 7: Moving a graph and dragging and dropping it back in the Main window

These examples do not exhaust the possibilities, of course: readers will be able to imagine many other kinds of productive interactions of these kinds, which have been made accessible by the interface in the device.

Measurement bar

Within the geometry applet on *ClassPad 300*, the measurement bar provides another opportunity for interaction. Essentially the measurement bar provides measurements of various aspects of figures that have been drawn, allowing the user to engage in any sort of exploration of figures and their properties. Figure 8 shows some examples for a right triangle that has been drawn, with the three screens showing respectively measurements of the slope of AB, the size of angle B and the area of Triangle ABC. The first screen illustrates that, in addition to a slope, the length of AB, the angle of inclination of AB or the equation of the line containing AB might all have been chosen. Thus, links between geometric objects and their metric properties are readily made through interaction with the screen.

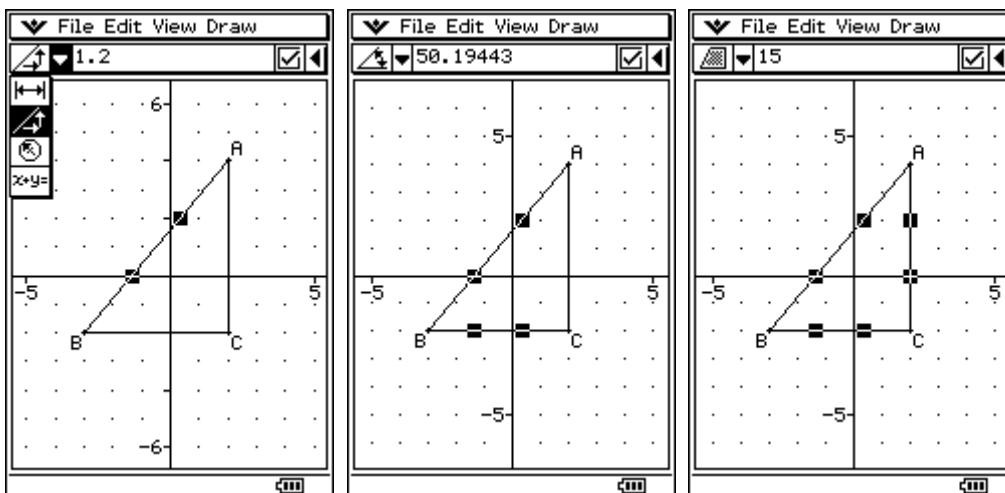


Figure 8: Making measurements using the measurement bar

The possibilities of productive engagement are further amplified by drag-and-drop interactions used in association with measurements. To illustrate this idea, Figure 9 shows some connections between equations, slopes and angles for the triangle from Figure 8. These were accessed using the measurement bar, and incidentally illustrate some opportunities for understanding numerical approximations. Both the equation and the angle have been dragged and dropped from the measurement bar to the Main window in this case.

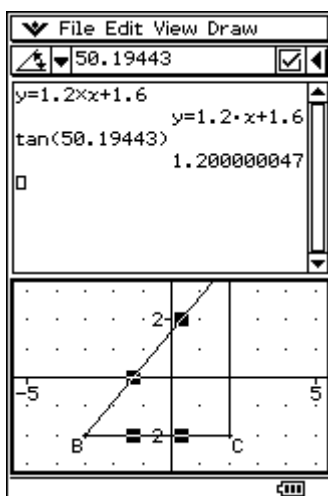


Figure 9: Dragging and dropping measurements into another window

Geometry link

Yet another kind of interaction involves a dynamic link between two windows; i.e., screens are automatically connected together, rather than relying on a drag and drop operation. This idea is accessible only within an *eActivity*, an electronic activity, designed to structure and direct a user's activity. (Kissane 2004). Figure 10 shows an extract from an eActivity concerned with equations for circles.

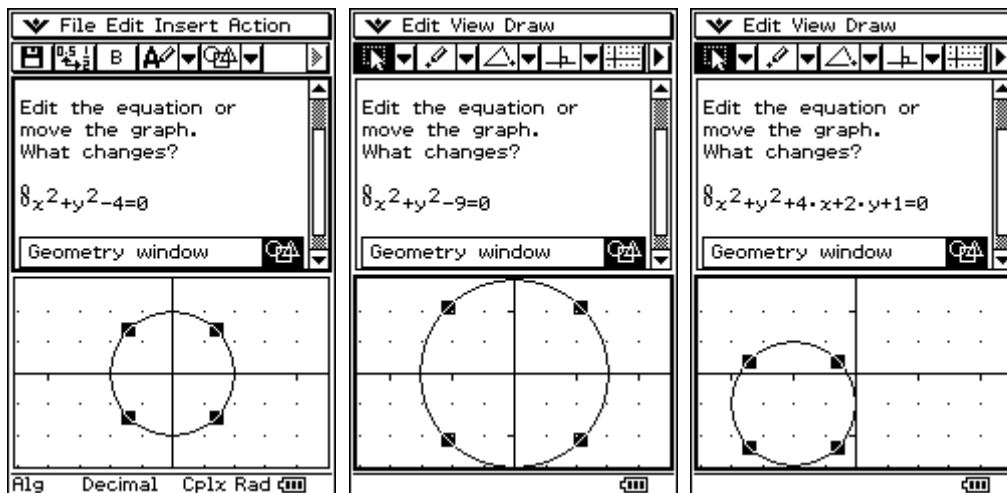


Figure 10: A geometry link inside an eActivity concerned with circles

The user of the eActivity (typically, a student with a *ClassPad 300*) can interact with this eActivity in two separate ways. Firstly, the linked equation ($x^2 + y^2 - 4 = 0$) can be edited, to see the consequences of changes. As soon as the editing is complete, the linked graph below changes accordingly. Alternatively, the graph itself can be moved by dragging it to a new position; as soon as this occurs, the equation is updated accordingly. The middle screen of Figure 10 shows an interaction of the first kind, while the third screen shows an interaction of the second kind. In this case, the eActivity has been set up so that the user can directly engage with these two elements in order to see how they are related, without having to know much about how to use the *ClassPad 300*. Interested readers can find many examples of eActivities on the Internet (e.g., HREF2).

Conclusion

Interaction is a key idea in current thinking about student learning of mathematics, partly because many of our best pedagogical practices focus on engaging students in activity and partly because we recognize that here are critical connections between different mathematical objects and ideas. Engaging students in productive interactions with, and between, mathematical ideas is important, so that technologies facilitating such interactions seem likely to grow in importance.

In summarizing his observations and some associated research studies with mathematical software of various kinds, Dreyfus (1994) concluded:

It is generally agreed that learning mathematics is not a spectator sport, but requires active involvement on the part of the learner; for learning abstract mathematical concepts, such activity is usefully described in terms of student actions on mathematical objects and relationships; these objects and relationships are necessarily given in some representation, which incorporates, or omits, links between them. ... computer tools have the potential to contribute to the learning process not only as amplifiers (saving time on computations and making graphing easier ...) but also, and more importantly, as reorganizers: Mathematics itself becomes different for the learner; new tools change cognition. Representations can be linked. Diagrammatic and qualitative approaches can be taken. (p. 210)

Although Dreyfus was writing before the invention of the *ClassPad 300*, his conclusions seem especially appropriate to this environment and offer some optimism for adapting it to educational use. In this paper, some possibilities opened up with the new *ClassPad 300* have been briefly illustrated. Translating such possibilities into effective classroom activities is not an easy matter, reminiscent of the observation many years ago by one of Charles Shultz's *Peanuts* cartoon characters: "There is no heavier burden than a great potential". However, the raw materials available in this device will lend themselves to many engaging interactions, whether the person using the stylus is a teacher or a student.

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- HREF1
<http://www.saltire.com/constraints.html>
- HREF2
<http://www.classpad.org/explorations.html>

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