Abstract

Over recent years, spreadsheets have been increasingly recognised as having a role in teaching and learning mathematics, but have continued to suffer the limitation of requiring access to computers. This paper summarises some of the potential benefits of spreadsheet use and briefly assesses some of the available research about it in schools. A spreadsheet application for a calculator is described and interpreted in terms of its capacity to handle the essential features of spreadsheets. The comparative merits of calculator and computer spreadsheets are discussed. Some ways in which calculator spreadsheets might be used for teaching and learning mathematics are outlined.

Introduction

While originally developed for the business world, spreadsheets continue to be used for purposes of mathematics education in secondary school. While this can be explained partly because of their widespread availability on personal computers in schools, homes and businesses, it is also a consequence of their particular suite of capabilities, which lend themselves to some kinds of mathematical work. Access to a spreadsheet has remained dependent on access to a computer, until fairly recently. In this paper, we first identify the features of spreadsheets likely to be of benefit to teaching, learning and doing mathematics, and then consider the prospects of a spreadsheet on a graphics calculator having significance for school mathematics.

Features of spreadsheets

This section describes briefly the key features of spreadsheets from a mathematical perspective, particularly with inexperienced readers in mind. Several useful links to augment this information considerably are provided on the web (Kissane, 2007). Some of these are exceptionally useful for inexperienced spreadsheet users.

Essential features

A spreadsheet comprises a matrix of cells that can contain either numerical or textual information. Data in the cells can either be entered directly by a user or can be calculated by the spreadsheet itself, with appropriate instructions from the user stored in the cell with a formula. The use of formulas allows some numerical values in a spreadsheet to depend on values elsewhere in the spreadsheet, effectively allowing a number of calculations throughout the spreadsheet to be completed simultaneously. Although other spreadsheets exist, Microsoft’s Excel is in such widespread use on
personal computers that it is almost universally relevant, and will be used here as the
archetypal spreadsheet, without further direct reference. Other spreadsheets (such as
those included in ClarisWorks and AppleWorks) are very similar at the level of basic
operations: differences appear only at the more sophisticated end, which is not of
particular interest in this paper.

Spreadsheet formulas generally look a little different from typical algebraic
formulas, although they have the same function. A spreadsheet formula uses a cell
address (typically a letter that references a column and a number referencing a row,
such as B3) to refer to the number in a cell, rather than the single letter characteristic of
algebraic representation (such as x). Spreadsheet formulas are recognisable in
spreadsheets as they start with an equal sign and refer to some other cells. Thus the
formula, =A1+3, stored in cell A2 would calculate three more than the value in cell A1
and store it in cell A2. In conventional algebraic representation, if the value in cell A1
was represented by x, this formula would calculate the value x + 3 and store this in cell
A2.

As well as direct arithmetical relationships, modern spreadsheets include a number
(in fact, a large number) of mathematical, statistical and other functions that can be used
in formulas and abbreviated ways of referring to sets of arguments of the functions. As
an elementary example, the formula, =AVERAGE(A1:A20), calculates the average of
the values in the twenty cells, A1, A2, A3, ..., A20. If this formula were stored into cell
A22, the spreadsheet would display the mean of the column of values in A1 to A20. If
any of the values in the column were changed, the mean would be changed accordingly
(when the spreadsheet is set to recalculate automatically). Formulas provide a means
of constructing mathematical relationships of various kinds, and thus generating them can
be thought of as a kind of programming.

A powerful feature of spreadsheets concerns filling a formula, which allows for
equivalent formulae to be constructed automatically by the software, rather than having
to be entered manually. To illustrate this, the average formula above could be readily
filled across into row 22 of columns B to D; the effect of this would be to automatically
generate the formulae to find the means of the first 20 values of columns B, C & D.
Although the formulas could be entered separately in each case, this facility allows an
efficient means of doing the same thing.

These two capabilities (formulae and filling) together allow a spreadsheet to be
readily used to study recursive situations in mathematics. The most obvious examples
are those concerned with recursively defined sequences, which can be represented
easily in a spreadsheet. The recurrence relation can be defined in terms of a formula (a
very simple case is the one described at the start of this section, using the formula,
=A1+1), and then filling this formula down a column. This process effectively repeats
the same relationship in successive cells. For this case, the filling down process will
define an arithmetic sequence with common difference of 3.

While the spreadsheet itself comprises an array of cells with text, formulae or
numbers in them, spreadsheets generally have mechanisms for representing the
information in more helpful ways, especially graphical ways. These essentially allow
the user to define a graphical representation (such as a scatter plot or a histogram) of
selected data from the spreadsheet itself.

Advanced features
The features described above as essential have been built into spreadsheets for many
years. Later versions of spreadsheets have included extra features that make them more
powerful, easier to use or otherwise more versatile in representing and handling numerical information. Space precludes an extensive description here, although it is important to acknowledge that advanced features are unlikely to be available on graphics calculators in the near future.

Some advanced features arise from the capacity of a computer to use a range of colours on powerful visual displays. Thus, conditional formatting features allow different cells to be coloured differently, depending on their content, which in turn allows for impressive visual displays of mathematical objects (such as the Sieve of Eratosthenes, Pascal’s triangle or a Sierpinski Gasket).

Other advanced features concern the range and sophistication of mathematical and statistical functions routinely available. Thus, successive versions of Excel tend to contain yet more mathematical functions, to meet the wide range of needs of users for calculations of various kinds.

Other features are concerned with designing user-friendly interfaces, with easy ways for users to input values into a cell (eg, to vary the values of a parameter in a relationship). Two methods used to good effect recently by Staples & Smith (2005) and Drier (2001) involve scroll bars and spinners. Such features are very useful for designing stand-alone spreadsheets for students to use to experiment with mathematical ideas.

### Spreadsheets in mathematics education

In the first issue of a new electronic journal devoted to the use of spreadsheets in education, Baker & Sugden (2003) surveyed the literature on the topic over 25 years, and concluded that “… there is no longer a need to question the potential for spreadsheets to enhance the quality and experience of learning that is offered to students.” (p.32) While some of the areas of educational use concern matters that are not directly related to mathematics, and others concern mathematical ideas not of concern to school mathematics, they highlight the considerable potentials of spreadsheets in some key areas. These areas include the elementary study of algebra, aspects of financial mathematics, statistics, numerical analysis and combinatorics (e.g., Abramovich & Pieper (1996)), among others.

Similarly, in reviewing published research in the area, Jones (2005) identified a number of aspects of algebra which researchers have claimed are supported by work with spreadsheets, and also recognised issues related to teaching and learning statistics. An important early study in this area was that of Sutherland & Rojano (1993), concerned with the development of early algebraic understanding using spreadsheets, in both the UK and Mexico. The spreadsheet offers an environment in which early ideas can be developed regarding functions and equations and ways of representing these.

One of the major attractions of spreadsheets, it seems, is that they are becoming available for use in a very wide range of settings, not only the mathematics classroom. Two important settings are those of the home and the workplace, in both of which personal computers are becoming much more common, and virtually ubiquitous in workplace office settings. These days, the bundled software for new computers usually includes a spreadsheet application, and this is certainly true for office computer systems. An application that is likely to be available so widely certainly deserves to be considered as a potential tool for teaching and learning mathematics.

Ruthven & Hennessy (2002) studied teacher accounts of their practices with ICT in a small number of English secondary schools and noted that spreadsheets were available in all of the schools studied; further, they found that in many cases students are more
likely now than previously to encounter spreadsheets in settings outside mathematics, so that some of the lost time getting started is dimishing. Such matters, of course, depend on the practices and facilities at particular schools to some extent.

In contrast, D’Souza and Wood (2003) studied Year 11 General Mathematics students in NSW, with a focus on the Financial Mathematics topic of the course. They found that in many schools, there were significant practical access issues related to the adequacy of computer resources and support and professional development associated with new curricula, which together reduced the success with which spreadsheets were used. In concluding their study, the researchers identified “… practical difficulties of the implementation and reasons for students’ resistance towards working with computers: too few working computers, computers not working properly, the time taken to learn the software, lack of computer confidence, etc.” (p.293) The authors note that these problems are not insurmountable, but nonetheless will impact on the success of innovations of the kind described.

Baker & Sugden (1993, p.24) suggest that a major advantage of spreadsheets for school mathematics is that they might save time for many activities, thus creating new opportunities for exploratory work of various kinds. Ruthven & Hennessy’s (2002) paper describes in some detail the kinds of pedagogical approaches which technology (not restricted or unique to spreadsheets) seems to support, at least in teachers’ reported practices. In circumstances more favourable than those referred to by some of the students in the sample of D’Souza and Wood, spreadsheets appear to offer promise.

Of additional interest is the extent to which software tools are permitted for student use in formal high-stakes assessment. Generally speaking, computers are not permitted in formal examinations in Australia, although there are opportunities for schools to permit their use for in-school assessments, which are significant for several school systems and states.

Increasingly, textbook authors seem to assume some level of access to spreadsheets, especially in mathematics courses related to finance. An early example of textbook use was Lowe et al (1994), who used spreadsheets as one environment in which algebraic relationships can be represented and equations solved. Recent textbooks published for the NSW General Mathematics course have included some work on spreadsheets, supported in part by formal requirements for students to complete a Computing Skills test, which includes work on spreadsheets, by the end of Year 10 in that state. In recent years, commercial publishers have included CD-ROMs with textbooks for secondary school, and these frequently contain examples of spreadsheets for students to use.

The development of some advanced features, described above, has permitted the production of spreadsheets designed as stand-alone objects to support mathematical exploration by students, demonstration by the teacher, or both of these activities. Good examples, using sliders and spinners in Excel, covering a wide range of senior secondary school mathematics topics have been provided by Staples & Smith (2005). Similarly, Drier (2001) has described a number of interactive Excel spreadsheets developed for use by teacher education students, some of whom have begun to use them in secondary schools as well. At a more sophisticated level, publications like that of Beare (1994) have focussed on extended examples of mathematical modelling, with significant support from pre-prepared spreadsheets designed for the purpose.

Modern spreadsheet software, such as Excel, now includes significant statistical capabilities, not only for descriptive statistics, but also for various inferential purposes, such as hypothesis testing. While these capabilities fall some way short of those of professional data analysis packages, they provide substantial data analysis opportunities for early undergraduate students of statistics, as well as those who work in the business
world, such as MBA students. The advantages of access in computer laboratories and workplaces (without additional software costs beyond those already invested), together with home use advantages, have increased the popularity of this use of spreadsheets in recent years. As noted by Nash & Quon (1996), who seemed to be not very enthusiastic about this use, there are significant criticisms of this practice, especially related to graphical capabilities and some issues of data handling. However, many Australian universities now use Excel routinely in the early years at least, because of the significant access advantages.

A graphics calculator version of a spreadsheet

Graphics calculators are clearly much less powerful than computers armed with sophisticated software, which is hardly surprising in view of the very great differences in costs of the two platforms for connecting technology and mathematics. What they lose in capabilities, they gain in access, as noted by many authors recently. For example, Ruthven & Hennessy (2002) reported the common practice in many schools of using hand-held machines, “… making it relatively easy for teachers to give students access to such technology in the normal classroom setting; sometimes also with projection facilities” (p. 58). To date, graphics calculators have not usually included spreadsheet applications routinely, although exceptions to this have been the very high end devices such as the Casio ClassPad 300 and the Texas Instruments TI-89. No doubt, one reason for this has been related to screen size and memory limitations. Another reason has possibly been related to the limited extra capabilities offered by a spreadsheet over and above what a modern graphics calculator provides.

The recent Casio fx-9860G AU calculator includes a spreadsheet as one of the standard applications, and so is here used to interrogate the potential value of hand-held spreadsheets in mathematics education. One part of the significance of ready access to technology is the flow-on effects of permission to use such technologies in formal examination settings; as noted above, this is rarely possible with computer spreadsheets.

As calculators have a much smaller screen than computers, less cells are visible on a calculator than on a computer, so that it is necessary to scroll more often to see the contents of cells. However, the same general cell characteristics are evident in both environments: cells contain text (for labelling), numbers (as data) or formulae (to define some relationships between cells). The syntax for formulae (e.g., starting with an equals sign and referring to cells by row and column) is similar to that for Excel, convenient for those already familiar with it. There are filling operations on a calculator, as for computer spreadsheets, as well as a range of mathematical functions, similar to those normally available on a graphics calculator, which generally accommodate the needs of secondary school students.¹

There are less statistical operations available in this calculator version of the spreadsheet than in computer spreadsheets. However, all the necessary statistical calculations, including those concerned with hypothesis testing, can be carried out as usual in the statistical area of the calculator; this is facilitated by the calculator commands to import and export data between the spreadsheet and the statistics or matrix areas of the calculator.

Calculator spreadsheets can be saved, edited and retrieved, as well as transmitted to other calculators or exchanged via the Internet. Of course, each might be constructed

¹ The operational details of the Casio fx-9860G AU spreadsheet are not fully described in this paper, but will be included in an associated workshop and are described in detail in Kissane & Kemp (2006).
instead by students themselves to explore a mathematical situation, rather than be prepared in advance by the teacher.

In summary, the key capabilities of spreadsheets described above are available via a hand-held calculator, while the advanced capabilities characteristic of recent software is not available in this environment.

**Some examples**

Clearly, space precludes an extensive catalogue of examples in this paper. Instead, three examples are offered to illustrate the kinds of work made possible by a spreadsheet on a calculator, and to consider this in light of the existing calculator capabilities. Each is briefly described here as an example of a prepared spreadsheet, to be used by students or used for demonstration purposes via a suitable projection device.

**Chance**

A spreadsheet can be used to provide experience of chance processes in an efficient way. For example, Figure 1 shows a spreadsheet called *Toss a Die* that simulates the tossing of an eight-sided die 200 times in Column A. The average result of the first 20 tosses is calculated and shown in cell B2, while the average of all 200 tosses is shown in cell C2. Every time the recalculate key is pressed, the entire spreadsheet is simulated again, with fresh averages displayed. Although rather slower than a computer, the calculator is still fairly fast, taking about two seconds to regenerate the entire spreadsheet each time. By inspecting the results of generating data in this way, students are able to see that there is a consistent pattern of results for the average of 200 tosses (usually close to 4.5), while the average for the first 20 tosses is less consistent.

![Figure 1: Successive results of tossing an eight-sided die](image)

A spreadsheet like this can help students get a feel for the nature of chance processes: while results are never certain, they are more predictable with larger amounts of random data available. Furthermore, issues related to sampling and sample sizes can be generated by comparing the stability of the averages of smaller samples.

In addition, changing the formula used in column A to generate the data allows for patterns connecting the die used with the averages obtained to be inferred and checked. Changing from an 8-sided die with all sides equally likely to occur to a regular $n$-sided die requires only a change from 8 to $n$ in the formula.

Simulations of this kind are also available on the graphics calculator through the STAT mode, but it is not possible to obtain immediately the means of the simulated die tosses. For each set of tosses, the mean would have to be separately retrieved, and it would not be possible to consider partial results, as was done in this case with the first twenty tosses. So the spreadsheet offers opportunities not otherwise available, and seems to augment the existing capabilities well.
Sequences and series

Spreadsheets are good tools for examining sequences and series numerically, and a relatively simple spreadsheet provides a mechanism for doing so. To illustrate this, two separate examples are provided here, one with an explicit definition of terms of the sequence and one with a recursive definition.

The middle screen of Figure 2 shows successive terms of the harmonic sequence, given by the reciprocals of the counting numbers and its associated series in a calculator spreadsheet called Series. The first screen illustrates the way in which the graphics calculator spreadsheet is designed to permit easy evaluation of a sequence in column B, when an explicit definition of terms is available. The corresponding series is obtained with standard spreadsheet formulae filled down column C, as suggested by the middle screen.

The third screen of Figure 2 illustrates how data from the spreadsheet can be readily graphed, using essentially the same mechanisms as those that apply for statistical data. In this case, the graph has been traced to explore the (lack of) convergence of the series.

Figure 3 illustrates the use of a spreadsheet called Fibonacci to explore a recursive sequence, in this case the Fibonacci sequence. The first screen shows how the two-step recursion is defined in a formula, which is filled down column B as far as desired. Column C is calculated as the ratio of successive terms of the sequence in Column B. These data can be graphed to show the (rapid) convergence of the ratios to the golden ratio, as shown in the middle screen.

This spreadsheet allows for easy variations on the theme to be explored in a What-if kind of way. For example, the spreadsheet begins with 1 in each of cells B1 and B2, which are needed to define the Fibonacci Sequence. The third screens shows that the change of these two cells to B1 = 3 and B2 = 4 results in the rest of Column B changing immediately. Inspection of the graph (not shown here, for space reasons) reveals the remarkable and quite unexpected result that the ratios once again seem to converge to the golden ratio.

In this case, the calculator already permits exploration of sequences through the RECUR mode. However, the spreadsheet introduces a capacity to quickly and efficiently explore the sequences and series; indeed, as noted by Baker & Sugden (2002), the spreadsheet saves time.
Finance

A common office use of spreadsheets involves financial planning of some kind. As noted above, D’Souza and Wood (2003) studied the use of computer spreadsheets in the Financial Mathematics component of General Mathematics in NSW. There are many ways in which elementary financial mathematics ideas can be represented on a spreadsheet for student analysis.

One example is spreadsheet Reducint, shown in Figure 4. This spreadsheet has been constructed to explore the concept of reducible interest, which it is necessary for students to understand as consumers, not only as mathematics students, since most practical uses of interest are of this kind. (Credit card statements and housing mortgage repayments are two examples of everyday significance to most Australian families.)

![Figure 4: Exploring reducible interest](image)

The first screen in Figure 4 shows how the spreadsheet has been set up to calculate monthly balances for a $5000 loan at 12% reducible interest and a monthly payment of $170. (The END balance after the final payment is displayed, to avoid needing to scroll to the bottom.). The second screen shows immediately the effects of a variation in interest rate to 16%, while the third screen shows the effect of increasing the monthly payment to $180.

Although calculations of these kinds can also be made in the calculator’s finance (TVM) module, the spreadsheet allows for all the reducing balances to be seen simultaneously and dynamically, in contrast to the less efficient TVM procedure.

Conclusions

Spreadsheets have acquired a place in the repertoire of technologies of significance for learning, teaching and doing mathematics over the thirty years since their invention in the early days of the personal computer. Part of their significance lies in their widespread availability inside and outside school environments. While recent embellishments of the basic concepts have improved their range of contributions to school mathematics, the essential features are valuable for constructing objects that allow students to explore some aspects of mathematics. The inclusion of these essential features on graphics calculators seems to extend the range of influence of the spreadsheet as a useful device for mathematics education in secondary schools, and is deserving of attention to exploit it appropriately.

References


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