Exploring space and measurement with the *ClassPad 300* *

Barry Kissane
Murdoch University, WA
B.Kissane@Murdoch.edu.au
http://wwwstaff.murdoch.edu.au/~kissane

Abstract: Hand-held technologies have offered less support for learning about space and measurement than they have for other strands of mathematics. This paper describes a recent development which seems promising in this respect, Casio's *ClassPad 300*. The mechanisms of providing interactive opportunities for students are described and illustrated, and some possible ways in which this sort of device might be used productively in mathematics learning are offered. A range of mathematical ideas are used, including geometric properties and relationships, coordinate geometry, transformations in the plane, mensuration and conics.

Introduction

More than a decade ago, *A national statement on mathematics for Australian schools* (Australian Education Council, 1990) identified five content strands for school mathematics, which continue to be used in curriculum frameworks around Australia. At around the same time, personal technologies in the form of graphics calculators began to be used in many schools. Since that time, the major uses of hand-held technologies in secondary schools have centred on the Number, Algebra and Chance & Data strands with much less opportunities concerned with the other two strands of Space and Measurement. Although there have been exceptions to this generalisation (such as the development by Texas Instruments of a version of *Cabri Geometry* for the TI-92 graphics calculator), the constraints of screen size have generally discouraged the development of suitable technology for dealing with spatial objects satisfactorily. This paper offers a brief description of the way in which the Casio *ClassPad 300* supports mathematics in these two strands, and considers some implications of this sort of device for school mathematics.

ClassPad 300

The *ClassPad 300* is a relatively new example of personal technology, about the same size as a modern graphics calculator, but with a larger graphics display screen and a smaller keyboard. The photograph in Figure 1 illustrates these two characteristics and also shows the main menu, from which various applications can be accessed.

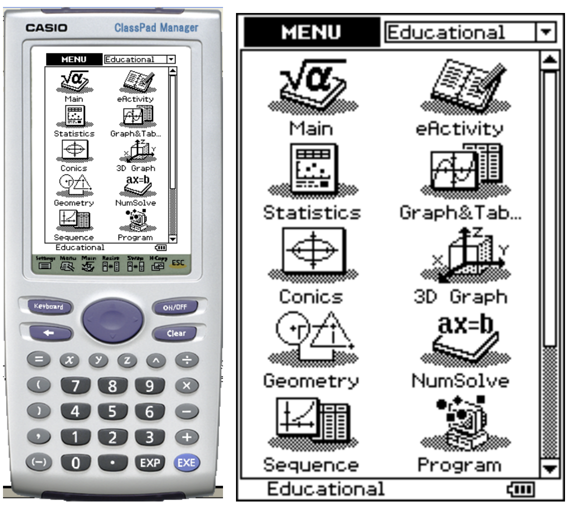

Figure 1: Photograph and opening menu of the *ClassPad 300*

As well as the keyboard shown in the photograph, the device has a touch-sensitive screen, which can be operated by a stylus, and a variety of soft keyboards. A PC-based emulator, called *ClassPad Manager*, is also available. This paper focuses upon the *geometric* resources provided by such an environment, the two main features of which are the personal nature of the technology (more portable and hence more accessible than, say, a conventional computer) and the use of a stylus for interaction between machine and person. Readers interested in details of the various other *ClassPad 300* capabilities can access them from the manufacturers (eg, Casio Computer Corporation, 2004), and may well find similar kinds of issues arising for other aspects of mathematics.

Interactive geometry

A major feature of the *ClassPad 300* concerns interactions between the user and the machine, made possible by the stylus. In the case of geometric work, the use of the stylus gives the experience a feel not unlike the use of physical manipulative materials or drawings, since there is a direct link between the screen elements involved (such as geometric objects) and the person using the stylus. Interactive experience of this kind is potentially very fruitful for learning, as argued in more detail in Kissane (2004). Similarly, Mason (1995, p.10) observed that “Screen-objects present a new form of apparatus or manipulable.”

Constraint-based geometry

Geometric objects are available on the *ClassPad 300* through the use of the Geometry application, visible as one of the icons in Figure 1. While so-called ‘dynamic geometry’ software such as *Cabri Geometry* and *Geometer’s SketchPad* has been used for some time in secondary schools, the *ClassPad 300* uses ‘constraint-based geometry’, described briefly by the software company responsible for it, Saltire Software (2004a). An earlier version of this sort of software seems also to have been used on a hand-held device by Hewlett Packard (Saltire Software, 2004b).

To illustrate the nature of ‘constraint’ in this context, the first screen in Figure 2 shows a parallelogram ABCD drawn on the graphics screen, using one of the pull-down icons on the Geometry toolbar. The vertices and sides of the parallelogram can be moved using the stylus, some possible results of which are shown in the second and third screens. Despite these movements, the figure ABCD remains *constrained* to be a parallelogram by the software. The defining constraints for a parallelogram used by the software ensure that opposite sides of ABCD remain parallel and congruent, regardless of how the vertices are moved or the dimensions and shape changed through the use of the stylus.

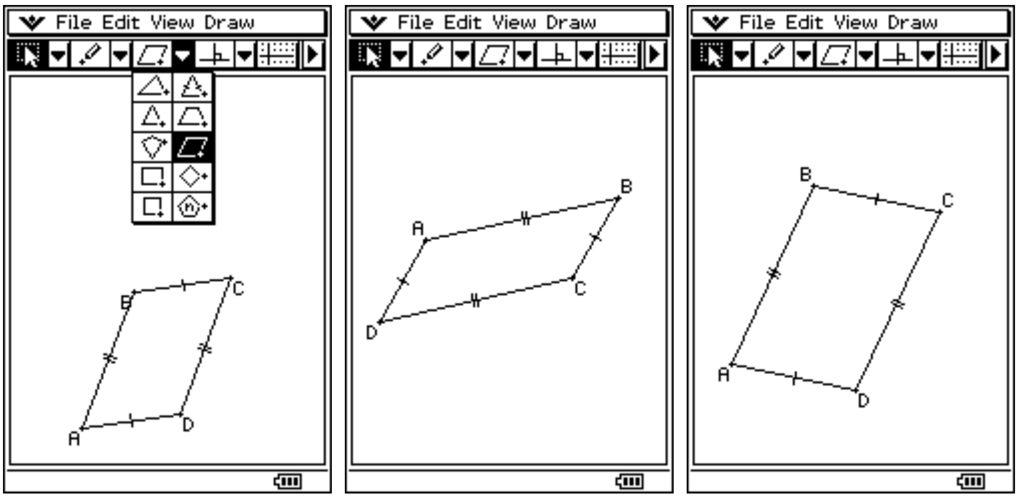

Figure 2: ABCD is constrained to be a parallelogram

Interacting with such a figure seems likely to help students get a good feel for the nature of a parallelogram and some of its essential properties, such as relationships between opposite sides, opposite angles, adjacent angles and so on.

Making measurements

In addition to the manipulation of screen objects, a second important feature of the Geometry application concerns the making of measurements of geometric objects, which might help students get a more quantitative sense of the properties of objects. In Euclidean geometry, we are interested in a range of measurements, such as those of length, angle, area, direction and so on. On the *ClassPad 300*, measurements are made via a 'measurement box', illustrated directly under the menu items in Figure 3. The first screen shows that the length of the selected segment AB is about 8.05 units. The second screen identifies the angle at vertex B by highlighting the adjacent sides AB and BC, and the measurement box shows the angle size of 103.0418°. Other possible measurements might be chosen from a pair of adjacent sides, as shown in the third screen; the measurement box indicates 'No' to the 'congruence checker', since the two highlighted line segments are not congruent.

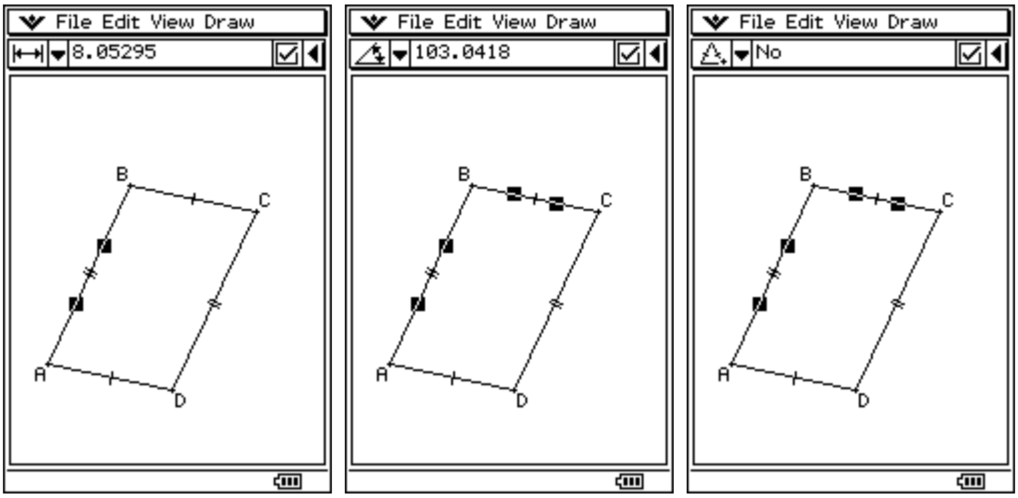

Figure 3: Some measurements of ABCD

In a similar way, different selections of parts of an object offer different opportunities for measurement and the resulting explorations. The first screen in Figure 4 shows that selecting all four vertices of parallelogram ABCD allows for either the area or the perimeter to be measured. The measurement box shows that the area is 41.03547 square units. The selection of only three vertices (which define a triangle, of course) allows for the areas of triangles DAB and ABC to be determined, as shown in the final two screens of Figure 4. In this case, these measurements help to see that each triangle has half the area of the parallelogram, or 20.51774 square units (within a rounding error), allowing students to verify that the diagonals of the parallelogram divide it into two parts of equal area.

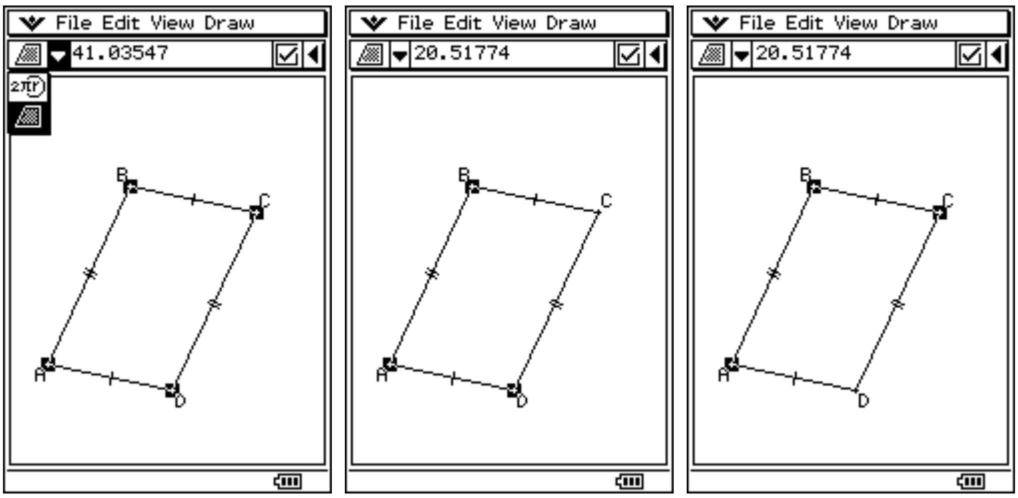

Figure 4: Measuring areas associated with parallelogram ABCD

Coordinate geometry

The measurements made in Figures 3 and 4 are relative to a particular scale of course, and the *ClassPad 300* allows for the rich connections between algebra and geometry to be explored by students, through the medium of coordinate axes. The first screen in Figure 5 shows the same parallelogram ABCD as in Figure 4, but with the coordinate axes revealed, as well as a grid marked to assist with locating objects in two-dimensional space. Although the measurement box is not showing, the first screen indicates that the coordinates of point C are (3.6,2.4), relative to the axes shown.

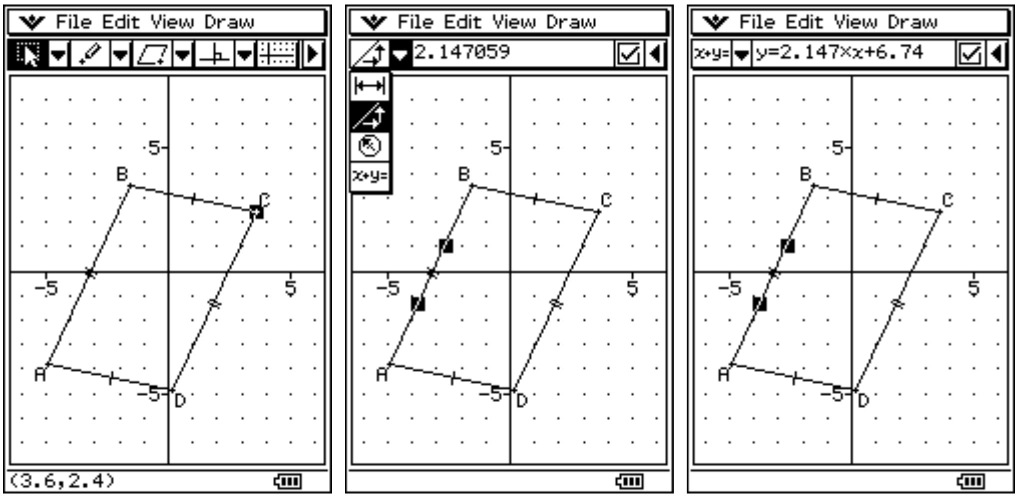

Figure 5: Revealing the coordinate system and making measurements

In keeping with Rene Descartes' wonderful invention, students now have access to algebraic ways of describing various geometric objects, as shown in the rest of Figure 5. The middle screen shows that one of the measurements of side AB is the slope of the corresponding line through A and B, which in this case is 2.147059. The drop down tools indicate that students might have just as easily chosen to measure the length of AB, the angle of inclination of AB with the horizontal axis or the equation determined by AB. The latter is illustrated by the final screen in Figure 5, with a clear link between the gradient of AB and the equation for AB.

There are many opportunities for students with these tools at their disposal to explore the properties of objects and to make the links between geometry and algebra that characterize coordinate geometry.

Geometric constructions

A common activity for students with with dynamic geometry software involves making geometric constructions and then exploring them to look for patterns, such as invariant properties. Many others have indicated the possibilities here for students to make discoveries for themselves, and to be thus encouraged to look for ways of proving that their observations are universally true (rather than relying on observations alone). Such activity is supported on *ClassPad 300*, since, as Saltire Software (2004) indicate, construction-based geometries are a subset of constraint-based geometries.

To illustrate this kind of learning opportunity, Figure 6 shows the use of a perpendicular bisector construction tool to show the three perpendicular bisectors of the sides of triangle ABC. Once the construction has been completed, the stylus can be used to move the vertices of the triangle to different locations, while the software ensures that the constructed perpendicular bisectors are moved accordingly. The middle screen suggests that these three are concurrent, while the third screen verifies that this seems to be the case, even when the three lines meet outside the triangle itself.

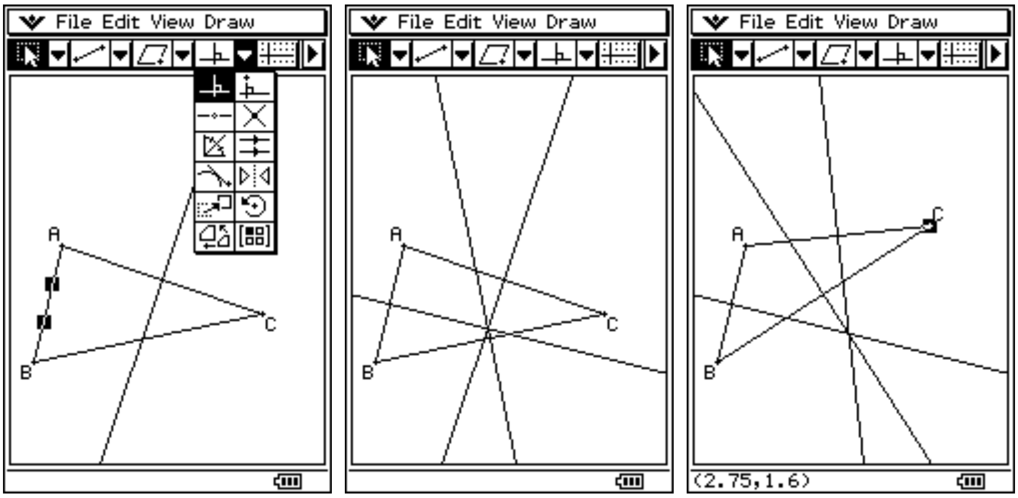

Figure 6: The perpendicular bisectors of the sides of a triangle are concurrent

Armed with such a tool, students can be given opportunities to notice and to explore spatial relationships in ways that have not been available prior to the development of dynamic geometry software. In this regard, Mason (1989, p.46) conjectured that "what is important about geometry is being aware of the fact that there are facts, rather than mastery of some particular few facts". While such activity does not provide the mathematical reassurance required for a proof, it hopefully provides a stimulus to look for reasons for observed regularities and to thus provide a need for proof.

Spatial connections

A distinctive feature of the *ClassPad 300* environment is that different applications can be connected together in educationally powerful ways. In this section, some promising examples of this facility are briefly described and illustrated.

Exploring functions

A powerful and popular use of graphics calculators has been to represent graphs of functions and to quickly explore relationships between changes in the parameters of functions and their graphs; indeed this idea is probably the first one used by mathematics teachers and continues to be a reason for describing the devices as 'graphing' calculators. Since it includes all of the functionality of a graphics calculator, a *ClassPad 300* can be used in the same way. To date, it has not been possible to do the reverse on graphics calculators: to see how changes in graphs are related to changes in the parameters of the functions concerned.

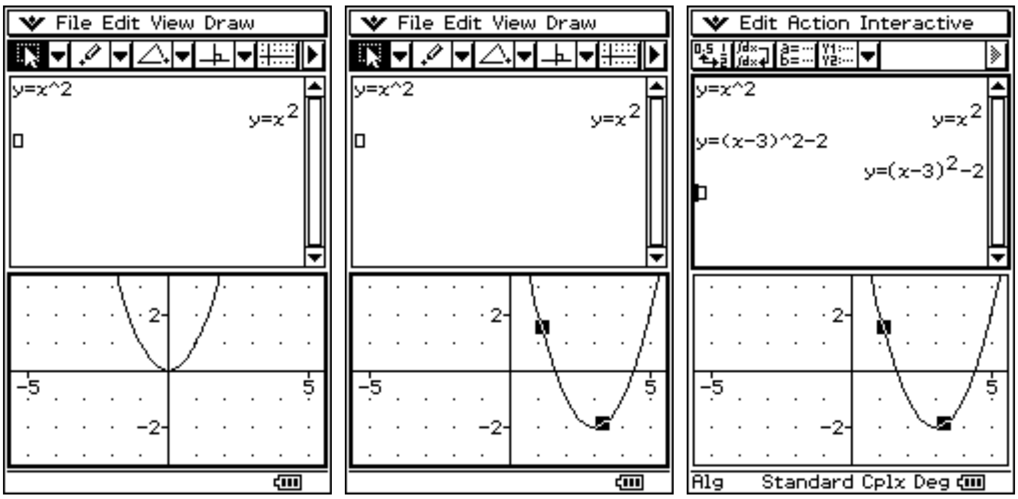

Figure 7: Dragging and dropping between algebra and geometry windows

Figure 7 shows a sequence of three dual screen with algebraic expressions in the top and a geometry screen at the bottom. In the first screen, the stylus has been used to physically drag the expression for quadratic function to the geometry screen, thus producing the expected graph. (An alternative involves cutting and pasting via the Edit menu, in much the same way as a word processor operates.) The second screen shows that the stylus has again been used to drag the parabola to a new position, three units to the right and two units down. Finally, the third screen shows the effect of dragging the (re-located) parabola back to the algebra screen, which results in the algebraic representation of the associated function. (Of course, it is a great deal easier to physically do these things than it is to describe them on paper.)

This sort of facility seems to offer much promise for students making sense of functions and their graphical and symbolic representations, as it complements the graphics calculator capability of moving from symbols to graphs very nicely. Armed with such a tool, students can experience at first hand the lovely relationships involved, and can see that they apply to various families of functions, not only quadratic functions. A limitation is that it is restricted to dragging, so that only translations (both horizontal and vertical, as illustrated in Figure 7) are involved; stretches and the corresponding scale changes do not seem to be able to be produced in this way, so that more conventional methods of exploring these remain important.

Exploring transformations

A second example of connections between geometric and algebraic ways of representing things concerns transformations in the plane. The *ClassPad 300*, like recent versions of geometry software, includes a facility for the isometries of reflection, translation and rotation, allowing these to be constructed and studied. The series of three screens in Figure 8 shows an example of a reflection about a line.

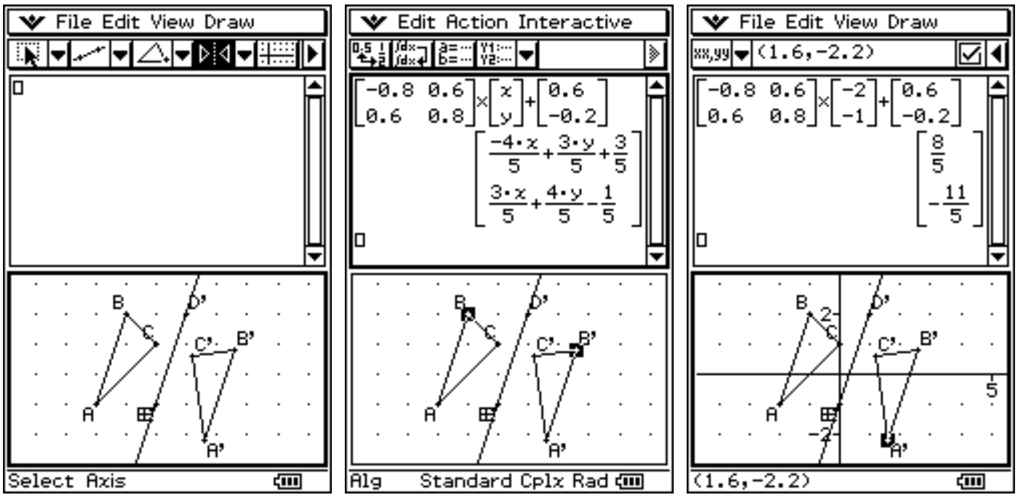

Figure 8: Exploring reflections about a line geometrically and algebraically

A reflection of the triangle ABC about the line DE has been constructed in the geometry window at the bottom of the first screen. The middle screen shows the results of selecting a point (B) and its image (B') and then dragging these together to the top window: the general linear transformation involved is then shown algebraically, using a matrix representation. The third screen includes the axes (which of course are needed to make sense of the matrix formulation) and also shows how the matrix representation can be used to find images of a particular point. In this case, the image of (-2,-1) after reflection in line DE is (8/5,-11/5). Again, it seems reasonable to expect that providing students with the facility to move smoothly in these ways between geometric and algebraic representations will offer learning opportunities that were previously unavailable.

Mensuration

Interactions between geometric measurements and the main screen allow for verification of mensuration formulas, as illustrated in Figure 9. In this case, the area of triangle ABC has been determined in four different ways, once directly in the measurement box and the other three using half the product of the lengths of a pair of sides and the sine of their included angle.

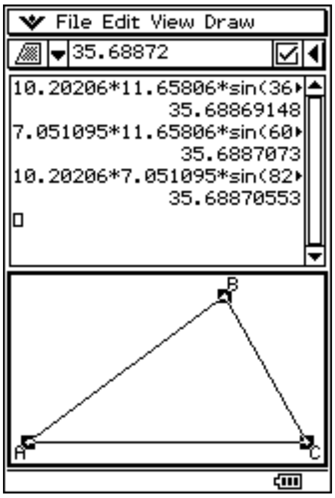

Figure 9: Finding the area of a triangle using two sides and the included angle

All necessary lengths and angles have been copied and pasted from the measurement box in the geometry window into the calculation window.(Measurements are of course rounded, resulting in slight inaccuracies in final decimal places.) What the *ClassPad 300* provides here is an opportunity to verify that formula produces equivalent results, even when different pairs of sides are chosen.

Dynamic linking

As a final example, complementary to the mechanism of dragging and dropping or cutting and pasting described above in the context of exploring functions, the *ClassPad 300* offers a means of dynamically linking algebraic and geometric objects, so that a change in a geometric object gives rise to a corresponding change in its algebraic representation and vice versa. This idea occurs in the context of 'eActivities', which are small user-created applications that are designed for students to experience various aspects of mathematics; an extensive discussion of these is provided in Kissane (2004) and there are many examples illustrated on the Internet (eg, at Saltire Software (2004c)).

Figure 10 shows an example of an eActivity. Each of the three screens shows a circle in the bottom geometry window and an associated equation in the top window. The first screen shows the initial situation. In the second screen, the equation parameters have been changed, resulting in the circle 'moving' and 'shrinking' accordingly. Similarly, the third screen shows that the circle itself has been dragged with the stylus to a new position, which has resulted in an updated version of the equation. In this particular case, as the centre of the circle has moved off the horizontal axis, the equation includes a linear *y* term, not previously evident. As for some earlier manipulations, it takes a good deal more words to describe this than to actually do it; again, it seems reasonable to expect that students given the opportunity to manipulate both expressions and geometric objects linked together dynamically in these sorts of ways will develop a strong sense of the connections involved.

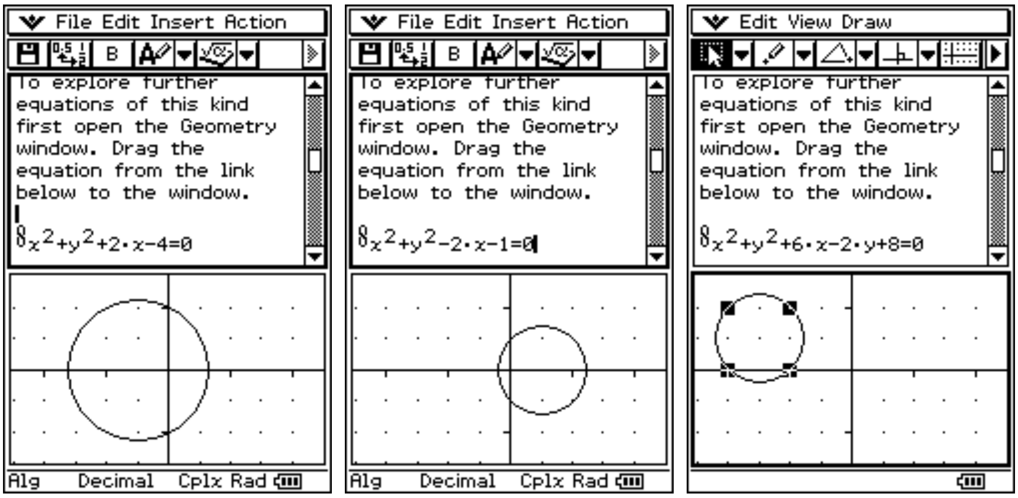

Figure 10: A dynamic link between circles and their defining equations

Conclusion

An important role of technology is to provide students with experiences that are not otherwise available to them. (Kissane, 2002). To date, the experiences offered by hand-held technologies seem most likely to be accessible to a wide group of students, but these have been relatively scarce in areas related to the Space strand and its connections elsewhere, such as the Measurement strand. Mason (1989, p. 46) argued that "… geometry takes place in a world of forms and images, entry to which is gained through the power of mental imagery, augmented and extended by dynamic images, drawings on paper and discussion with colleagues". Technologies of the kind described here seem to offer new opportunities for enhancing mental images in students, through the design of productive classroom activities of new kinds, based on personal interaction between mathematical ideas and their representations. While taking advantage of these new opportunities is unlikely to be an easy matter, and it is too early to tell whether the educational effects are as promising as they at first seem, the design of new technologies like this offers important opportunities to explore these important issues in school classrooms.

References

Australian Education Council (1990) *A national statement on mathematics for Australian schools*. Melbourne: Curriculum Corporation.

Casio Computer Corporation (2004) *ClassPad 300*. Retrieved 29 August 2004 from http://classpad.net/.

Saltire Software (2004a) Constraint based geometry. Retrieved 30 August 2004 from http://www.saltire.com/constraints.html.

Saltire Software (2004b) *Math Xpander*. Retrieved 30 August 2004 from http://www.saltire.com/xpander.html.

Saltire Software (2004c) *ClassPad explorations*, Retrieved 31 August 2004 from http://classpad.org/explorations.html.

Kissane, B. (2002) Three roles for technology: Towards a humanistic renaissance in mathematics education. In Rogerson, A. (ed) *The Humanistic Renaissance in Mathematics Education: Proceedings of the International Conference* (pp 191-199). Palermo, Sicily: The Mathematics Education into the 21st Century Project.

Kissane, Barry (2004a), Interactive mathematics on the *ClassPad 300*. In B. Tadich, S. Tobias, C. Brew, B. Beatty & P. Sullivan (eds) *Towards Excellence in Mathematics*, (pp 259-269), Melbourne: Mathematical Association of Victoria.

Kissane, Barry 2004b. The design and use of eActivities for learning mathematics. In W.-C. Yang, S.-C. Chu, T. de Alwis & K.-C. Ang (eds) *Proceedings of 9th Asian Technology Conference in Mathematics: Technology in Mathematics* (pp 367-376), Singapore: ATCM Inc.

Mason, J. (1995) What, why and how in mathematics. *Micromath*, 11(1), 10-14.

Mason, J. (1989) Geometry; what, why, where and how? *Mathematics Teaching*, 129, 40-46.

*This paper was presented to the 20th Biennial Conference of the Australian Association of Mathematics Teachers, January 2005, and is published in the Conference Proceedings:

Kissane, Barry 2005, Exploring Space and Measurement with the *ClassPad 300*. In M. Coupland, J.Anderson & T.Spencer (eds) *Making Mathematics Vital*, (pp 324-332), Sydney: Australian Association of Mathematics Teachers.