Using the HP 38G in upper school: preliminary thoughts

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The following examples have been devised to show some of the range of ways in which the HP 38G graphics calculator might help students with the mathematical ideas often encountered in upper secondary school. They are not meant to (and certainly do not) exhaust all the possibilities. rather, they are a sampling of the *kinds* of activities that seem appropriate in upper secondary school mathematics courses.

Examples in boxes are intended to be read by teachers, and are not necessarily written in a language designed for students. They assume some prior familiarity with the operation of the HP 38G.

1.   Functions

Students can draw graphs easily to study relationships between functions and their graphs, once they are familiar with how to enter functions, adjust the ranges, read the scales and manipulate graphs.

Graph $Y1 = X^2$, $Y2 = (X - 1)^2$, $Y3 = (X - 2)^2$ and $Y4 = (X+1)^2$. Compare the graphs.

Then graph $Y1 = \sin X$, $Y2 = \sin(X-1)$, $Y3 = \sin(X-2)$ and $Y4 = \sin(X+1)$. Compare the graphs.

Compare the graphs between sets. What is common?

Define a function as F1(X).
Then define F2(X) = F1(X) +3.
Define F2(X) = 2*F1(X)
Press the graph button and compare the three graphs.
Then change the first function, and graph again.

Define a function as F1(X).
Then define F2(X) = F1(X–2) and F3(X) = F1(X+1)
Press the graph button and compare the three graphs.
Then change the first function, and graph again.

Define two functions F1(X) and F2(X), one linear and the other nonlinear.
Examine the graphs of F1(F2(X)) and F2(F1(X)).

Graphs of functions can be analysed to answer questions of interest about the functions.

> Find the minimum value of $F(t) = 2t^2(t – 3)$ for positive values of the variable t.
>
> Define $F1(X) = 2X^2(X–3)$, and then adjust the axes using the RANGE or ZOOM commands. Then use the minimum command on the FCN menu.

2.   Data analysis

Data can be edited and stored for long term use. Numerical statistics are readily available. Graphical displays are available.

> Data analysis is completed using the statistics Aplet. Delete all previous data before entering new data. Enter a univariate data set of your choice.
> Once data are entered, statistics are available through the STATS key, and can be graphed with PLOT .

Graphical statistical analysis, especially scatter diagrams, help interpret correlation coefficients. Various regression lines can be drawn on the scatter diagrams.

> These data show the air temperature at various heights above sea level on a recent commercial flight into Perth.
>
> | Height (metres) | 5793 | 5640 | 4169 | 3994 | 2534 | 2297 | 1251 | 920 | 409 | 20 |
> |---|---|---|---|---|---|---|---|---|---|---|
> | Temperature (^{o}C) | -17 | -15 | -4 | -2 | 8 | 9 | 11 | 14 | 17 | 18 |
>
> Describe the relationship between height and temperature. Use your description to predict temperatures at some new heights.

> Data transformations, in Statistics symbolic view

> Log linear analysis

3. Growth and decay

Recursive relationships can be studied by using a command line repeatedly.

E.g., Investigate the sequence given by $T_0 = 4$; $T_{n+1} = 3T_n - 1$, n 1.
Press $\boxed{4}$ $\boxed{\text{ENTER}}$ to enter the initial term.
Then $\boxed{\times}\boxed{3}\boxed{-}\boxed{1}$ $\boxed{\text{ENTER}}$ to get the next term. Notice how the calculator uses *Ans* (which refers to the previous answer) to complete this calculation.
Now each successive press of $\boxed{\text{ENTER}}$ generates a further term of the sequence, as the recursive command line is repeated each time, with the value of *Ans* changed.

Recurrences can be used to determine the *n*th term of a geometric or arithmetic sequence and of an arithmetic sequence, for small values of *n*.

For example, Press $\boxed{11}\boxed{\text{ENTER}}$ to enter the initial term.
Then $\boxed{+}\boxed{2}$ $\boxed{\text{ENTER}}\boxed{\text{ENTER}}\boxed{\text{ENTER}}$ ... gives 11, 13, 15, 17, ... , or
$\boxed{\times}\boxed{3}$ $\boxed{\text{ENTER}}\boxed{\text{ENTER}}\boxed{\text{ENTER}}$ ... gives 11, 33, 99, 297, ...

Sequence Aplet.

Some problems of geometric growth and decay can be addressed using log-linear graphs.

Graph F1(X)$=3.5e^{0.4X}$ and F2(X)$=ln$ (F1(X)) to see the effect of the transformation. .

Compound interest is easiest to deal with as a recurrence relationship.

To find successive annual amounts from investing $800 at 9% per annum compound interest, press 800 $\boxed{\text{ENTER}}$ and then
$\boxed{\text{X}}$1.09$\boxed{\text{ENTER}}\boxed{\text{ENTER}}\boxed{\text{ENTER}}$ ... .

4. Equations and inequalities

Graphical methods give good approximations, by repeated tracing and zooming, or automatically.

Use a graph to solve $x^3 - 2x + 1 = 3$.

Now use the same graph to solve $x^3 - 2x + 1 < 0$

How many solutions are there to the equation: $2 \cos x + 1 = \dfrac{x+1}{2}$?

Solve simultaneously the two equations:

> Y1 = 1000 ln(t + 1.2)
> Y2 = 60 + 200t

by first defining them, and then adjusting the axes.
Set -1 < X < 15 and -100 < Y < 3000, and then use the intersection command on the FCN menu.

The Equation Solver (in the Solve Aplet) automates numerical methods of solution by finding the value of a variable after all others have a value.

Enter a formula for the volume of a cylinder into the solver: $a = \pi r^2 h$. Press ⌈ENTER⌉ then give values to r and h, pressing ⌈ENTER⌉ after each. Press ⌈ENTER⌉ to indicate a value of a is sought.

Repeat a few times to 'solve' for other variables.

Linear systems can be solved using matrices. Here is an illustrative example, using easy numbers so that a mental check is possible.

Solve the system: 2x + y = 17
                        x – 2y = 2

Define matrices M1 for the coefficients and M2 for the result vector. The solution is given by M1^{-1}*M2

Of course, realistic data and larger systems are handled with comparable ease.

5.   Randomness

The HP 38G has a random number generator, which can be used in various ways.

The rolling of a die can be simulated with int (6*RANDOM+1). Each time ⌈ENTER⌉ is pressed, a new roll of the die is simulated. Both the integer part and the random command (int) are available on the MATH menu, in the REAL and PROBABILITY menus respectively.
    A Bernoulli trial with p = 0.6 can be simulated with the command:

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INT(RANDOM+0.6)
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The random number command gives a number uniformly distributed on (0,1). Each time ⌈ENTER⌉ is pressed, a new trial is simulated, with 1 representing 'success' and 0 representing 'failure.

6.   Calculus

Limits can be explored graphically. Of special importance is 'local straightness' — the idea that any continuous function is linear over a sufficiently short interval. This is evident on repeatedly zooming in on a graph at a point. Standard limit results are also accessible in new ways:

> Graph the function F1(X) = sin X/X and zoom in near X = 0.

Numerical derivatives are available on the HP 38G. The slope command, accessed from FCN, gives the derivative at a point, once a graph is drawn. Derivative functions can be defined and graphed. These are powerful tools for learning calculus concepts.

> For example, try F1(X) = cos X and F2(X) = dx (F1(X)). (dx is accessed in the CHARS menu)

Asymptotic behaviour can also be addressed visually.

> Examine the asymptotic behaviour of $f(x) = \dfrac{x^2 + 4}{x - 1}$ or $g(x) = \dfrac{x^3 + 4}{x - 1}$ by graphing the function and then zooming out several times or manually adjusting the axes suitably. Explain what you notice.