

Using the HP 38G in lower school: very preliminary thoughts

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The following examples have been devised to show some of the range of ways in which an HP 38G graphics calculator might help students with mathematical ideas often encountered in lower secondary school. They are not meant to (and certainly do not) exhaust all the possibilities, and they have not all been field-tested with young students; they represent plausible speculations.

Examples given in the boxes are intended to be read by teachers, and are not necessarily written in a language suitable for young students. They assume some familiarity with the operation of the HP 38G. They may well be better communicated orally than in writing, in several cases.

In most cases, students will gain more from working on activities of these kinds in pairs or in small groups, rather than individually, so that they have someone with whom to discuss their actions, observations, predictions and expectations; they will also have someone to help them use the calculator. In many cases, a calculator shared between two students is adequate for productive work.

1. Plotting points

Clear any functions left (by a previous user) in the Symbolic menu before starting.

Points can be plotted directly when in graph mode by using the **PLOT** menu, which is accessed by **2nd** **ZOOM** **B** **1**. Enter the X and Y coordinates respectively, followed by **ENTER**.

If the default axes are chosen, students can see how points can be made to appear in each of the four quadrants (or appear in none of them if the ordinates are too big!) Activities like the following will help beginners see how the coordinate system works, as well as how the calculator works, and get them accustomed to interpreting the screen (which is critical to sound graphics calculator use).

Another advantage of the default axes is that the units on each axis are the same length.

Plot a number of points (at least five of them) with X coordinate equal to 2. Explain what you see.

Then do the same with a number of points with Y coordinate 3.
Plot the point (2,1). Now plot three more points so that the four points are at the vertices of a square. Make sure it looks like a square, and then explain to your partner how you know you are right. Plot some more points on the sides of the square.

Plot the vertices of a square with sides 2.5 units long and one vertex at (1,3). In how

many *different* ways can you do this?

Plot the four vertices of a square that doesn't have vertical sides.

Plot several points for which the X coordinate and the Y coordinate are the same. Describe what you see. Predict where on the screen the points (1.4,1.4) and (-2.6,2.6) will be.
Now plot some points for which the X coordinate is one more than the Y coordinate.
Make some other rules like these and plot some points. Explain what you see to your partner.

Notice that line segments can be plotted. In turn, similar activities to those above can be devised.

2. Number sequences

Recurrences can be used to determine successive terms of a sequence for which a rule is known, or has been hypothesised, for small values of n .

For example, press $\boxed{5}$ $\boxed{\text{ENTER}}$ to enter an initial term of 5.

Then $\boxed{+}$ $\boxed{2}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$... gives 5, 7, 9, 11, ... , or

$\boxed{\times}$ $\boxed{10}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$... gives 5, 50, 500, 5000,

The mechanism for these operations is that the effect of the command $\boxed{\text{ENTER}}$ is to execute the current command line. If no new command is entered, the previous one is merely repeated. In these cases, the value of the variable *Ans* is changed each time.

Press $\boxed{4}$ $\boxed{\text{ENTER}}$ to enter an initial term of 4.

Then $\boxed{+}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$... gives 4, 7, 10, 13, ...

What will be the 8th term in this sequence? The 50th term? the 1000th term?

Will the sequence include the value 127? What about 250?

Find a rule for this sequence.

Don't forget that decimals and fractions are part of the number system too. Activities like the following may help students see how these numbers are represented and related:

Use fraction mode.

Press $\boxed{1}$ $\boxed{/}$ $\boxed{3}$ $\boxed{\text{ENTER}}$ to enter an initial term of $\frac{1}{3}$.

Then $\boxed{+}$ $\boxed{1}$ $\boxed{/}$ $\boxed{6}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$... gives $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \dots$

Press $\boxed{7}$ $\boxed{\text{ENTER}}$ to enter an initial term of 7.

Then $\boxed{-}$ $\boxed{0}$ $\boxed{.}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$... gives 7, 6.9, 6.8, 6.7, ...

Students can also use their calculators to plot data points directly, as suggested above, with an eye towards seeing if there are any patterns among them. Make sure that they set the axes suitably before they start, since later changes will erase any points already plotted.??

3. Memories and variables

Graphics calculators use alphabetic memories. On the HP, they are labelled in pink (A to Z) and are accessed with the A..Z key (hold it down) and $\boxed{\text{STO}}$. Once values are stored in the memories, they can be recalled by using the memory name, which exactly mimics the idea of a variable as a placeholder that can have a variety of values. When manipulated on the screen, the normal conventions of algebraic notation are followed. In particular, juxtaposition refers to multiplication. Thus $\boxed{\text{A..Z}}$ $\boxed{\text{A}}$ $\boxed{\text{A..Z}}$ $\boxed{\text{B}}$ $\boxed{\text{ENTER}}$ gives AB, the product of memory A and memory B. Other more complex expressions like $A(B + C^2)$ are as readily constructed and remain faithful to the conventions of algebraic notation.

Give A the value of 5, using $\boxed{5}$ $\boxed{\text{STO}}$ $\boxed{\text{A}}$ and give B the value of 7 with $\boxed{7}$ $\boxed{\text{STO}}$ $\boxed{\text{B}}$.

Use the $\boxed{\text{ALPHA}}$ key to recall the values of A and B.

What is the value of AB?

Change the value of A to 9, and use the calculator to find AB again.

make some more changes to the values of A and B to see how the calculator interprets AB.

4. Linear functions

Students can draw graphs to study relationships between linear functions and their graphs, once they are familiar with the coordinate system, how to enter functions, adjust the ranges, read the scales and manipulate graphs. Many kinds of explorations are available before students begin to use the more sophisticated graphical explorations tools in the $\boxed{\text{ZOOM}}$ and $\boxed{\text{FCN}}$ menus. A few examples are given below.

Draw a graph of the rule given by $Y = X - 1$. Then use $\backslash\text{x}(\text{TRACE})$, $\backslash\text{x}(X,Y)$ and the cursor keys ($\boxed{\blacktriangleleft}$ and $\boxed{\blacktriangleright}$) to trace the graph. How do the X and Y coordinates of points on the graph compare? Explain why.

Compare the graphs of the following rules, all drawn at once: $F1(X)=X$, $F2(X)=X+2$, $F3(X)=X-2$. What is the same about the graphs? What is different about them?

Compare the graphs of the following rules, all drawn on the same screen: $F1(X)=2X$, $F2(X)=2X+1$, $F3(X)=2X-2$.

What is the same about the graphs? What is different about them?

Make some more examples like this for yourself.

Draw a graph of $F1(X)=2X+13$ on the standard screen. Explain what you see. Use the Plot Setup key to see different parts of the graph.

Draw some linear graphs through the point $(0,1)$. How many can you draw?

Draw a graph of $F1(X)=2X-1$. Trace the graph to see if there is a point for which $Y=4$. Use your observations to solve the equation $2X-1=4$. Check that your solution is correct.

Draw a graph of $F1(X)=2X-1$ and another graph on the same axes of $F1(X)=5-X$. Find the coordinates of the point at which the graphs intersect. Explain to your partner how these coordinates could be used to help you solve the equation $2X-1=5-X$, and check that the solution is correct.

How could you use a graph to solve the equation $x - 2 = 7 - 2x$? How many solutions would you expect there to be, if any?

Discuss your views with your partner before graphing to solve the equation.

Give rules for at least three graphs that are parallel to the graph of $F1(X)=2.7+1.3X$. Check using the calculator that the graphs do satisfy the condition.

5. Data analysis

The significant difference between a scientific and graphics calculator is that the data are stored in graphics calculators, while only *summaries* of the data are generally retained in scientific calculators. So data can be checked, edited and retained for long term use. Numerical statistics are readily available as are graphical displays.

Data analysis is completed using the statistics Aplet.

Once data are entered, statistics are available through the **STATS** key, and data can be graphed with Plot.

6. Mathematical properties

Because the calculator faithfully represents the real number system and its properties, explorations allow students to find out for themselves some of these properties. (A similar

remark is true for both four-function and scientific calculators, although these do not always have such nice screen features as the HP 38G.)

Perhaps the best example concerns indices, which are readily constructed using the $\boxed{x^y}$ key. The laws of indices and sensible meanings for negative and fractional indices are not difficult to discover in this environment, and demand that students think about *why* the relationships hold. Appropriate kinds of computational activities include the following:

Evaluate $2^3 \times 2^4$, and express the result as a power of two. Examine $3^2 \times 3^3$ similarly as a power of three. Try some other products of powers of the same base.

How can you predict the result without having to work it out on the calculator?

Test your predictions with your partner.

Evaluate $(5^2)^3$. Is it the same as $(5^3)^2$? Why?

Write $(5^2)^3$ and $(5^3)^2$ as powers of 5. Explain your results to your partner.

Try some other examples like this.

Evaluate 4 to the power one half. Then evaluate 9 to the power one half and 3025 to the power one half. What do you notice in each case? Try some more examples to test your observations until you are confident that you can see what is happening.

Predict the result of evaluating 3 to the power one half, and tell your partner what you predict. Then use the calculator to check.

Raise some numbers to other fractional powers and try to find out what the result means. For example, what is the effect of raising numbers to the power three quarters?

Raise some numbers to *negative* powers. Start with 2^{-1} and 5^{-2} . Explain the results.

8. Transformations

For students who already have the rudiments of graphing functions, including entering functions, adjusting the ranges, and reading scales, the calculator provides rich opportunities to compare graphs of related functions to see the effect of linear transformations. For example, in the case of parabolas, activities like the following can help develop visual imagery.

Students can draw graphs easily to study relationships between functions and their graphs, once they are familiar with how to enter functions, adjust the ranges, read the scales and manipulate graphs.

Graph together each of the four functions $Y1 = 3 - X^2$, $Y1 = 2 - X^2$, $Y1 = 1 - X^2$, and $Y1 = -1 - X^2$. How are the graphs similar? How are they different? Explain the similarities and the differences.

Now predict what the graph of $Y1 = 3 - X^2$ will look like. Sketch your prediction and explain it to your partner. Then graph it to test your prediction.

Repeat for $Y1 = X^2$, $Y1 = (X+1)^2$, $Y1 = (X+3)^2$ and $Y1 = (X-2)^2$.

Graph $Y1 = X^2$, $Y2 = (X - 1)^2$, $Y3 = (X - 2)^2$ and $Y4 = (X+1)^2$. Compare the graphs.

Then graph $Y1 = \sin X$, $Y2 = \sin(X-1)$, $Y3 = \sin(X-2)$ and $Y4 = \sin(X+1)$. Compare the graphs.

Compare the graphs between sets. What is common?

Define a function as $F1(X)$.

Then define $F2(X) = F1(X) + 3$.

Define $F2(X) = 2 * F1(X)$

Press the graph button and compare the three graphs.

Then change the first function, and graph again.

Define a function as $F1(X)$.

Then define $F2(X) = F1(X-2)$ and $F3(X) = F1(X+1)$

Press the graph button and compare the three graphs.

Then change the first function, and graph again.

At appropriate stages, similar kinds of activities can be used for other kinds of functions, such as reciprocal and exponential functions.

9. Randomness

The HP 38G has a (pseudo-) random number generator, in the MATH menu, which can be used in various ways to build some intuition about random processes. The rolling of a die can be simulated with the command: `ipart (6*random+1)`, but this requires some knowledge of the `ipart` function, which gives the integer part of a number. (The `ipart` command is given by `MATH` `3`). A sequence of activities like the following might help students to understand this.

Generate a random number by pressing `RANDOM` in the `MATH` menu. The number will always be between 0 and 1. Press the `ENTER` key a few times to get some more.

Make a screen full of random numbers and compare the results with those of other students.

Generate a random number by pressing `6RANDOM` Now the number will always be between 0 and 6. Press the `ENTER` key a few times to get some more. Make a screen full of random numbers and compare the results with those of other students.

Repeat using `6RANDOM +1`. What has changed? Why? Make a screen full of random numbers and compare the results with those of other students.

Use the `FLOOR` operation a few times to see what it does. For example, try `FLOOR 4.6` (in the `MATH` menu). Try `FLOOR 4.778` and `FLOOR 3.4532` as well. Explain what `FLOOR` does to a number.

Then repeat the earlier experiments using `FLOOR(6RANDOM +1)`.

Explain your results. Make a screen full of random numbers and compare the results with those of other students.

In what ways are your results like tossing an ordinary die?

Repeat using `8RANDOM +1`.

What has changed? Why?

Make a screen full of random numbers and compare the results with those of other students.

In the classroom, productive comparisons might be made of the results among pairs or groups of students. (E.g., simulate the tossing of a die 20 times, and count how many sixes each person gets. Compare them. How different are they? How similar? Make a table of results for the whole class. What happens in the long run? Why?)

More systematic experiments may be conducted by using the data analysis functions in the HP 38G. For example, enter `RANDOM` instead of a number for each element of a new data set. Then the mean and other statistics are readily available for inspection. ??