

Evaluating normal probabilities on a graphics calculator*

Barry Kissane
Murdoch University
<kissane@murdoch.edu.au>

One of the strengths of a graphics calculator is the potential it offers for students to explore mathematical ideas from a few different perspectives. A good example of this involves the normal probability distribution, frequently encountered in senior secondary courses involving statistics. This note describes a few ways in which the Casio cfx-9850G series of calculators can be used to evaluate normal probabilities.

Prior to the availability of graphics calculators, students generally had to refer to tables of the cumulative standard normal distribution in order to evaluate probabilities. In most (but not all) cases, the tables available provided the area under the curve between $z = 0$ and $z = 3$ or more, so that significant work is required to deal with cases other than the most direct ones.

Standard normal probabilities can be evaluated numerically in RUN mode of the cfx-9850G series, using the relevant commands in the OPTN > PROB menu. The screen below shows the three commands available, $P(t)$, $Q(t)$ and $R(t)$.



Once the PROB menu is showing, you'll need to press F6 to display these menu choices.

The symbols refer to the following, in which z is a standard normal random variable:

$$\begin{aligned}P(t) &= \text{Prob}(z < t) \\Q(t) &= \text{Prob}(0 < z < t) \\R(t) &= \text{Prob}(z > t)\end{aligned}$$

In the first instance, the calculator provides a worthy substitute to looking up the relevant table, as the examples in the next screen show for the particular case of $t = 0.6$:

P(0.6)	0.72575
Q(0.6)	0.22575
R(0.6)	0.27425

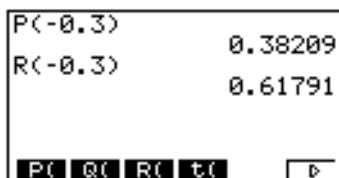
It is clear from this example that $P(t) = Q(t) + 0.5$ and that $P(t) + R(t) = 1$, a result that students can easily find out holds in general, not just for this case.

These commands provide more than is provided by a table, however, since they avoid the need for some of the (complicated) juggling of areas under the normal curve that are needed for less straightforward probabilities, such as those in which z is negative.

The screen below shows two of these. Previous methods of evaluating such probabilities usually have required the use of standard normal tables and the symmetry properties of the normal distribution:

$$\begin{aligned}\text{Prob}(z < -0.3) &= 1 - \text{Prob}(z < 0.3) \\ \text{Prob}(z > -0.3) &= \text{Prob}(z < 0.3)\end{aligned}$$

On the calculator, however, these can be evaluated *directly*, using $P(t)$ and $R(t)$ commands:

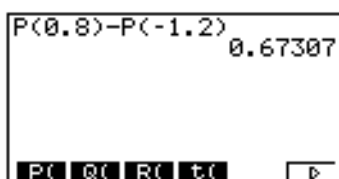


However, the cases that have generally caused students most problems in the past are those that involve a probability that the standard normal variable falls between two particular values. For example, to find from tables the probability that z is between -1.2 and 0.8 would require calculations such as the following:

$$\text{Prob}(-1.2 < z < 0.8) = \text{Prob}(z < 0.8) - \text{Prob}(z < -1.2) = \text{Prob}(z < 0.8) - [1 - P(z < 1.2)]$$

$$\text{i.e. Prob}(-1.2 < z < 0.8) = \text{Prob}(z < 0.8) + \text{Prob}(z < 1.2) - 1$$

It is easy to make a small slip with such calculations, with disastrous consequences. However, on the calculator, the probability can be computed *directly*, as shown below:



We have frequently encouraged students to sketch a suitable normal curve when evaluating probabilities, mostly to offer a quick rough check on the evaluation using tables and also to help constructing the complicated relationships between a desired probability and those offered in a table. In fact, some student drawing templates have included a standard normal curve for precisely this purpose. Similar sketches can be drawn very quickly on the cfx-9850G series, using the Sketch menu in RUN mode in association with the above commands from the OPTN > PROB menu. An example is shown in the next two screens.



The left screen shows the command needed in RUN mode, while the right screen shows the results, drawn on a window chosen automatically by the calculator. The command **Graph Y =** is obtained with these key strokes:

SHIFT F4 (Sketch) F5 (GRPH) F1 (Y=)

As for a hand-drawn sketch, the calculator graph provides some quick visual reassurance that the result seems plausible, based on areas under the standard and familiar bell curve. Students may be interested to notice that the scales used are not equal, but have been stretched considerably vertically, as can be readily seen by looking at the view window:

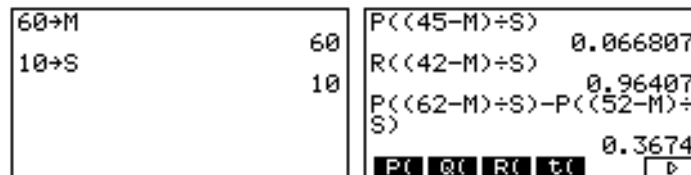


Tick marks on the x -axis are at every unit, while those on the y -axis are at every 0.1 units. (I suspect that many of us as teachers had not realised previously that such a drastic vertical stretch was needed to produce what we came to regard as a familiar shape.)

Non-standard normal distributions require a linear transformation to allow evaluation of probabilities using standard normal tables. Specifically, if x is distributed normally with mean μ and standard deviation σ , then we use the transformation

$$z = \frac{x - \mu}{\sigma}$$

Appropriate z -scores can be obtained manually in such a way, so that the calculator can then be used as before. Alternatively, the calculator can be left to do the calculations, which is convenient if there are several of them. The screen on the left below, refers to an example of a random variable normally distributed with mean $\mu = 60$ and standard deviation $\sigma = 10$; for convenience, these two parameters have been stored in memories M and S respectively, with multiple calculations in mind.



The screen on the right shows three separate calculations, respectively $\text{Prob}(x < 45)$, $\text{Prob}(x > 42)$ and $\text{Prob}(52 < x < 62)$

Normal probabilities can also be computed directly as integrals, using the definition:

$$\text{Prob}(z < x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

In the case of the standard normal distribution, $\mu = 0$ and $\sigma = 1$, so that the expression is a little less complicated:

$$\text{Prob}(z < x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{x^2}{2}} dx$$

The screens below show this function entered and graphed in GRAPH mode, using a slightly different view window as that shown above, in order to have a more friendly version.



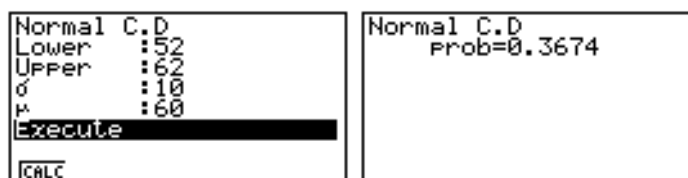
To evaluate a probability using this graph, use the G-SOLVE command for numerical integration. For example, the screens below show the use of G-SOLVE here to evaluate Prob $(-1.5 < z < 0.5)$



The view window chosen has the effect that each press of the trace key moves 0.05 in the horizontal direction, allowing this example to be accurately estimated.

A similar process can allow the general case to be handled, although the function concerned is very complicated and requires considerable care in entering.

Finally, yet another means of evaluating normal probabilities is available to those with a Casio cfx-9850GB PLUS model. This model has a Distribution menu (DIST) in STAT mode, one of the possibilities being the cumulative normal distribution. The screens below show the same result as previously



In summary, the calculator offers several ways of evaluating probabilities, each of which provides a different perspective for students coming to grips with the ideas of a normal distribution and its relationships with the standard normal distribution. With careful design, these different possibilities can be used to good effect to both evaluate probabilities efficiently and to understand some of the relationships involved.

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