TECHNOLOGY AND THE CURRICULUM:
THE CASE OF THE GRAPHICS CALCULATOR

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This paper outlines some of the complex relationships between the curriculum and technology, with particular attention on graphics calculators. Graphics calculators are of key significance for practical reasons of availability and affordability Until technology is available, curricula are not likely to be adjusted to accommodate them. Particular features of graphics calculators, including their recent extensions to algebraic calculators, are highlighted, and their significance for the curriculum evaluated. Curriculum change elements are interpreted in the light of calculators: the political process, materials development, assessment structures and professional development.

Introduction

The past decade has seen a number of technology conferences for mathematics teachers, mathematicians and mathematics educators in a number of countries. At the recent ICME-9 in Japan, the Working Group for Action concerned with technology attracted more participants than any other single group. Conferences of teachers generally have a large proportion of sessions concerned with the use of technology of one kind or another in mathematics education. It is clear that issues of technology are prominent in the minds of many of us, not only those of us who choose to attend a conference such as this one for which technology is clearly a major focus. This paper offers a brief outline of some of the relationships between technology and the mathematics curriculum, coloured somewhat by my experiences in Australia and the USA, and mindful that these relationships, like technology itself, are somewhat fluid. Partly because of my experiences, partly because of my interests, but mostly because of its central importance at this time, the focus is on the particular case of the graphics calculator, rather than other technologies. This is not to suggest, however, that similar kinds of analyses are not appropriate elsewhere.

What Technology?

As this conference amply demonstrates, there are many kinds of technology considered relevant to school mathematics these days. These range from very powerful computer systems, such as *Mathematica* and *Maple* to much less powerful, but much more pervasive, technologies such as those involving paper and pencil. Although it is worth remembering that some traditional mathematical practices such as the deployment of the long division algorithm or the use of a table of critical values for a particular statistic are examples of technologies, the focus of this paper will rest upon *electronic* technologies in general and the graphics calculator in particular.

The developments of new technologies relevant to school mathematics over the past decade have been quite astonishing. In more prosperous countries, the microcomputer has developed from being an expensive device occasionally found in classrooms and sometimes found in laboratories in school to being a device that is common in many households. Computers have become smaller, easier to use and much less expensive. Some (few) schools and universities have even mandated the use of laptop computers for students. Well-constructed software for educational purposes, such as *Cabri Geometry*, *Geometer's Sketchpad* and *Fathom* have appeared, along with increasingly powerful computer algebra systems, comprehensive statistical packages and powerful spreadsheets such as Microsoft's *Excel*. Communications technologies such as the World Wide Web and electronic mail have moved from scientific teams in universities to the everyday world of many adolescents in their schools and homes. Such developments are exciting, and it is an important function of a conference such as this one to probe their significance for mathematics education.

The choice of graphics calculators is motivated mainly by the potential for them to be available to essentially all students all of the time, rather than to some students all of the time, to all students for some of the time or, worse, to only some students for only some of the time. In this respect, I regard them as an example of what Schumacher referred to as "... *intermediate technology* to signify that it is vastly superior to the primitive technology of bygone ages but at the same time much simpler, cheaper and freer than the super-technology of the rich." (1974, p.128). Many studies, in many (developed) countries over the past decade have reached similar conclusions regarding the penetration of computer technologies into the everyday world of mathematics classrooms. Recent local examples include Thomas (1999) who concluded that New Zealand mathematics teachers had limited access to computers for educational purposes and Norton (1999) who noted that, even when access problems were minimised, many mathematics teachers in Queensland made relatively little use of computers. A further testament to the significance of graphics calculators is the recent national conference of the Australian Association of Mathematics Teachers (Morony & Stephens, 2000), which took place in direct response to a mounting professional view that this form of technology was so critical to the various curriculum deliberations around Australia that nothing short of a national summit would suffice to allow for the free interchange of ideas, experiences, concerns and plans for the future.

Graphics calculators have been commercially available for longer than compact disk players, so it should not be necessary to describe their attributes in fine detail. The key physical features, however, are that they are small, portable, battery-powered and have a multi-line display on which graphic objects can be drawn. Operationally, they are programmable, with a constant memory and have a number of inbuilt mathematical capabilities. In the least sophisticated cases, these capabilities include all those of scientific calculators, together with data analysis and function graphing; in more sophisticated cases, the capabilities also include numerical equation solving, matrix arithmetic, complex number arithmetic, recursion and elementary hypothesis testing. Importantly, graphics calculators from the various manufacturers have now reached their third generation, and so are all relatively easy to use. Arguably, graphics calculators are the first example of an electronic technology specifically designed for education, mathematics education in particular. In the last few years, affordable algebraic calculators have appeared, best regarded as graphics calculators with the embellishment of relatively unsophisticated computer algebra systems (Kissane, 1999), thus further blurring distinctions between 'computers' and 'calculators'.

What Curriculum?

Although there are many similarities, which more than justify the value of international meetings such as this one, the mathematics curriculum differs in content and structure between countries. Importantly, however, the curriculum has many forms within a country or, as in Australia, where curricula are state-owned rather than national, within a state.

Cuban (1992) distinguishes at least three levels and types of curriculum. The *intended* curriculum usually takes the form of a written document and has an official status. It describes what students are expected to know, understand and be able to do, and the circumstances under which these things will happen. In so doing, it represents a kind of consensus on what aspects of mathematics are most important. A second level concerns the *implemented* curriculum, which describes what actually happens when teachers attempt to deliver the intended curriculum to real students in real schools. It is frequently strongly affected by the materials such as textbooks and the classroom practices of teachers. Finally, the *attained* curriculum refers to what students actually learn as a result of being in the classroom. The attained curriculum is strongly related to what is tested and refers to the curriculum from the perspective of the learner. Each of these levels is important and deserves to be an object of study in its own right. Many research studies over recent years have suggested that there is a good deal of slippage between the ideals of the intended curriculum, the realities of the implemented curriculum and the attained curriculum that results.

Two strong, and related, school mathematics curriculum trends of the 1980's and 1990's are towards 'mathematics for all' and towards mathematics as a 'useful' subject. Even a casual glance at modern western curriculum materials such as student textbooks will reveal an apparent concern to highlight the applicability of mathematics to situations outside mathematics and clear attempts to broaden the appeal of mathematics to as wide an audience as possible. Both of these have some significance for the role of technology, and particularly for graphics calculators.

Partly because of the bewildering array of other things to choose, it now seems that the overwhelming majority of mathematics students opt for mathematics with a view to using it for some other purpose. In such a circumstance, an emphasis on a very practical view of mathematics, with less concern than many of us would like for aspects of mathematical thinking such as deductive reasoning and proof, is not unexpected; the availability of technologies that handle many of the practical and computational aspects of mathematical work is critically important. While studying mathematics for its own sake has never been a popular activity, it seems even less so nowadays, so that almost all students stop studying mathematics as soon as they are no longer obliged to opt for it. My estimate in the case of Western Australia, for example, is that much less than 1% of an age cohort of secondary students will ever study tertiary mathematics beyond the minimum requirements for their chosen profession. Almost all teaching of mathematics at many Australian universities, including mine, is described as 'service teaching', to meet the needs of graduates in other disciplines. Perhaps paradoxically, however, there are still many school mathematics curricula that seem to be framed on a premise that most students will pursue mathematics beyond the minimum.

Curricula and Calculators

Until quite recently, technology was not regarded as a major influence on the mathematics curriculum. Although some official curricula acknowledged the invention of computers, there were few significant attempts to adapt mainstream secondary school mathematics curricula to technology before the 1990's, as Waits & Demana (2000) and several others have noted. In contrast, the recent revision of the NCTM *Standards* (2000) has highlighted technology in a 'technology principle':

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning. … Students can learn more mathematics more deeply with the appropriate use of technology. … In

> mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics. The existence, versatility and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how best they can learn it.(pp.24-25)

The *Principles and Standards* suggest that learning can be enhanced by technology in a number of ways: more examples can be accessed by pupils than formerly, more time is available for conceptualising, multiple perspectives are accessible and feedback is provided in novel ways. Teaching is enhanced because teachers can use technology to provide experiences to students that would not otherwise be available, such as mathematical modelling using probability simulation. Both what is taught and when it is taught are affected. For example, optimisation of functions can be treated prior to the formal methods of the calculus and more complex symbolic manipulations can be dealt with using an algebraic calculator than with paper and pencil methods.

An Enabling Technology

In exploring relationships between calculators and curricula, it seems natural to start by asking what can graphics calculator do? Earlier, a brief and incomplete catalogue of typical mathematical capabilities was given. What sorts of things do these *enable* for mathematics education?

In the first place, graphics calculators enable various kinds of guided explorations to be undertaken, which would have been too difficult without technology. As an example, pupils can examine directly whether or not a particular sequence appears to converge, the effects of changing parameters of a function on the shape of its graph, and the significance of outliers on various sample statistics. They can explore the relationships between gradients of pairs of lines and the lines themselves, the consequences of varying interest rates, the effect on a histogram of changing the widths of the intervals and the connections between equations and graphs. Enthusiastic claims have long been made for exploratory activity in mathematics; graphics calculators enable some of this to be realised in classrooms.

Secondly, graphics calculators can handle some–indeed many–of the routine, computationally intensive, aspects of elementary mathematical work. Examples include the solution of quadratic or simultaneous linear equations, testing an hypothesis involving two sample means or evaluating a definite integral. The mathematical concepts underpinning such procedures are rich and important for understanding. Yet all too frequently in the past it seems that pupils without calculators devote a great deal of time to calculation and correspondingly less to making sense of it. A shift in the balance of attention to concepts and skills might be possible and certainly would be desirable.

Thirdly, graphics calculators can offer new opportunities for pupils to encounter mathematical ideas not presently in the curriculum. One example is the use of simulation as a means of tackling some situations involving uncertainty. Another involves the use of iterative techniques to study some of the mathematics of chaos. Another involves dealing directly with real-world data collected electronically and transmitted to a calculator. Still others involve the use of elementary programs to study repetitive processes.

Empirical Evidence

A good deal of work has already been done by researchers and practitioners to study the effects of using graphics calculators in various ways. As might be expected in changing circumstances, some of the early work is located in a previous curriculum and educational context, with the graphics calculator externally imposed. Thus, in their excellent review of research up to that time, Penglase & Arnold (1996) note the methodological concern:

> The current state of research into the use and effects of graphics calculators, then, remains inconclusive. Few studies distinguish carefully between the use of the tool and the context of that use. Claims regarding the relative effectiveness of the tool are frequently based upon assessment procedures which equate "student learning" and "achievement" with performance upon traditional tests, and fail decisively to account for important influences upon attitudes and conceptual understanding. (p.82)

Ruthven's (1995) analysis also repays careful study. He suggests that we need to look closely at how pupils actually use calculators (including graphics calculators).

> [A]t the level of principle, many important issues surrounding calculator use remain poorly conceptualised: our understanding would benefit from renewed curiosity, and a readiness to build connections with wider issues and other technologies. … In many settings, calculators offer the most realistic prospects of transforming classroom mathematics within the medium term so as to incorporate considered use of computational technology. (p. 464)

There is not space in this paper for a detailed and critical review of recent research using graphics calculators, although such a review is now needed. However, recent work may redress some of the concerns expressed by Penglase & Arnold, and move in directions that Ruthven would agree are productive, as attention shifts to curriculum settings in which the calculator is better embedded. A good recent example is the study by Doerr & Zangor (2000), who identified a number of different (and mainly productive) ways in which pupils made use of graphics calculators in practice over a considerable time period with a competent teacher. An interesting finding of the study was that the small screens on graphics calculators tended to inhibit collaboration between pupils, while overhead projector versions of calculator screens encouraged powerful forms of classroom discussion and debate.

Empirical work of this kind will be important to accumulate and assess in the near future.

What is Now Important?

Until quite recently, there was no alternative to many of the analytical procedures associated with school mathematics. To find the stationary points of a function, differentiation was unavoidable. To find the solutions to a cubic equation, factorisation was unavoidable. But this is no longer the case. With a graphics calculator available, good enough numerical approximations to stationary points can be found visually, while numerical solutions to the cubic can be obtained quickly. In addition, the analytical solutions of such tasks can be supported by an algebraic calculator. The difficult curriculum decisions now involve deciding under which circumstances these new facilities should be ignored, whether or not they should be banned, and whether (and when) we should help pupils to use them. As Kennedy (1995) has argued so eloquently, a good deal of what is in the secondary school curriculum used to be necessary for any pupils who wished to progress further in mathematics. But it is less clear now that this is the case, and the time has come for a careful re-evaluation of what we have come to take for granted.

It seems inevitable that a reconsideration of what is important and central about mathematics ought to take place as technologies such as calculators become available. It is a reflection of the inertia and the natural conservatism of school curricula that they seem to be so resistant to change of this kind. Written partly in frustration, but mainly in a spirit of improving the mathematics curriculum, Ralston's (1999) paper concerned with the abolition of paper and pencil arithmetic offers a strong case, almost a quarter of a century past the introduction of affordable arithmetic calculators to schools.

The development of algebraic calculators, of no less significance for the secondary school than are arithmetic calculators for the primary school, suggests a need for further analyses of this kind; hopefully, this will not take another quarter of a century (Kissane, 1999). Examples of early analyses of what algebraic manipulations are important to be done by hand (or by head?) and what can be well left to a calculator are provided by Bjork & Brolin (1998) and Herget *et al* (2000). Similarly, Goldenberg (2000) offers a tentative first list, suggesting that the mathematics education community ought be discussing such things now, if we are to find the necessary common ground from which to move forward. In presenting his list, Goldenberg notes:

> Why would anyone care whether students could solve problems like [those] above without using a symbolic calculator? After all, the calculator will do that stuff for them! I would make the case that one cannot make intelligent use of the technology without having reasonable fluency with some such skills *without* the technology. The reason is that it is hard to know what computations to ask for without understanding what the computations will *do*. (p. 14)

Arcavi (1994) has provided a very nice analysis of 'symbol sense', worthy of careful study in order to help decide which aspects of algebra ought to occupy our attention most urgently. Thinking of this kind, together with empirical evidence of the kind likely to be available from the University of Melbourne project (McCrae *et al*, 1999; Stacey *et al*, 2000) will help to reach the sort of agreement Goldenberg seeks. One of our problems appears to be a lack of communication between teachers of mathematics at different levels. The resulting mismatches of expectations and emphases are problematic for pupils and deserve more attention.

Aside from the details of what is better left to machines and what is appropriately done otherwise, it seems clear that pupils need to learn how to use graphics calculators effectively, as part of the mathematics curriculum. Effective use involves much more than knowing about the detailed steps needed to operate a particular calculator, of course. It also includes learning to make good decisions about when to use and when not to use a calculator. It involves learning how to interpret calculator output with the right mixture of skepticism and acceptance. Such matters ought to be a conscious part of the implemented curriculum, as they are much too important to be left to chance. While some have expressed concern that we should be teaching mathematics and not technology, a view I share, it is still important that students learn enough about the technology to use it well. This is no different in principle from the need to teach pupils how to use tables of square roots, calculations via logarithms or deft use of a slide rule in days gone past. What has changed is that the technology has become more complex, pervasive and powerful, thus requiring more effort to master. Of course, only the most optimistic would expect that pupils will develop the critical discretionary expertise in situations in which calculators were often not available to them, or in which their use was controlled entirely by decisions of others, such as a teacher, a textbook or an examination paper.

Curriculum Change

Adjustments to an official curriculum seem unlikely to happen until technology is (or can be) pervasive. A major reason for the significance of the graphics calculator is that it offers the best prospect for this to happen at the moment, certainly somewhat greater than is the case for the computer, the lap top or the Internet. Statutory authorities responsible for curriculum decisions are understandably likely to be very cautious at taking curriculum revision seriously in the light of technologies to which only some students have good access.

The Change Process

It is worth noting that curriculum change does not usually happen as a direct result of empirical research studies, but rather through a process involving a number of influences (among which, ideally and critically, research conclusions are one). Indeed, it is rare that empirical research provides an unequivocal, unambiguous direction for curriculum developers. Instead, the most critical functions of research are frequently to provide new perspectives for thinking about the curriculum and new questions to address. It is vital that good research be sponsored, conducted and reported, but naïve to expect that research results will be sufficient evidence upon which curriculum change will be built.

Statutory authorities legally responsible for making decisions about the official curriculum generally seek advice from a range of stakeholders, appropriately. At the senior secondary school level, this range is likely to include a mixture of tertiary mathematicians, secondary school mathematics teachers, representatives of key school systems and others (such as representatives of industry, professional societies and even parents). Perhaps curiously, and probably unfortunately, it is relatively rare for mathematics educators, researchers or curriculum developers to have much voice in official decision-making processes. By definition, such a grouping of people will make political decisions, based on the real and perceived power structures among them. Until recently, in many parts of the world, university mathematicians have traditionally been the main voices in curriculum decisions about secondary school mathematics, probably because a major goal of senior secondary mathematics is to prepare pupils for further study of mathematics and because university mathematicians are regarded as more qualified to identify what is most important about mathematics at present. When graphics calculators are considered, this presents a particular problem, as Tucker (1999) has noted:

> [M]ost college or university mathematicians have spent no time at all with a graphing calculator and are not inclined to spend the start-up time of an hour or two to learn, especially since they are unlikely to use graphing calculators on a regular basis outside the classroom. A bridge is needed for this gap between mathematics students (and secondary school teachers) on the one side and college faculty on the other. (p.910)

As is well known, relationships among those with curriculum responsibilities have not always been harmonious, and seem to have been especially problematic in recent times in some quarters. The case of California and the so-called 'math wars' have received prominence, indeed notoriety, but there are tensions of a less public kind also to be found elsewhere, including Australia, when issues related to the place of technology in the curriculum are debated. Indeed, as noted earlier, we need to have more debates, even at the risk of exposing our differences.

Change processes of this kind are challenged with the very significant problem of the rate of change of technology. Put simply, the rate of change of graphics calculator technology realistically available to secondary school pupils is a good deal greater than the rate of curriculum change possible. While enthusiasts are understandably irritated by slow responses to exciting new opportunities, conservatives (or merely the less-enthused) are understandably anxious about changing too much too quickly, so that decision-making bodies advised by both kinds of people, among others, are unlikely to make changes that keep abreast of changing technologies. Some, such as Podlesni (1999) have suggested that calculator R & D teams may well have an undue influence on curriculum development in such circumstances:

> Are we getting to the point where technology companies are making de facto curriculum decisions for us? Are they paving the way, consciously or unconsciously, for their future leadership in that process by making calculators upgradeable–through their software, one presumes? … Are we doing our job as teachers or relinquishing part of it to the electronics industry? Are we becoming unpaid salespeople for that industry with every new model? (p. 89)

Technology companies are necessarily driven mainly by the commercial market place rather than sound educational thinking and planning, even when they seek and obtain advice from people in the field. While such influence is mainly problematic, it can also serve to force us to confront big issues earlier than we would otherwise be likely to do. The best example of this is the development of algebraic calculators in recent times. Although computer algebra systems have been around for a good while now, it is their availability on hand-held devices, produced by commercial companies, that has demanded that our profession consider the complex issues involved with some urgency.

Curriculum Materials

As noted above, the curriculum materials provided by or to the teacher are a significant influence on the implemented curriculum. Ideally, a calculator-sensitive curriculum would be supported by appropriate materials. In fact, this is much more difficult to do than is commonly recognised, for a number of reasons.

In the first place, it is not an easy matter to produce good quality materials that integrate technology into the fabric of a course. Curriculum development projects such as the University of Chicago School Mathematics Project in the USA and Nuffield Advanced Mathematics in the UK have produced good exemplary materials, using the expertise of large teams of people. Such activity is very important to provide examples of what is possible, opinions about what is desirable and, hopefully, evidence about what are the consequences of using such materials in practice. However, many curriculum materials are produced with less resources and less access to specialist help and thinking. In Australia in recent years, for example, textbooks for secondary mathematics have begun to accommodate graphics calculators, but usually in the form of 'revised' versions of previous textbooks. In some cases, the revisions seem more concerned with appearance than with substance, as publishing companies are understandably anxious to be recognised as providing 'current' materials. Adding the occasional calculator screen dump or calculator activity does not constitute a sound form of revision. There seems to be no particular reason for expecting authors of textbooks written without consideration of technology, graphics calculators in particular, to be particularly capable of the necessary re-thinking. These circumstances are exaggerated by commercial imperatives to appear in the marketplace before the competition does.

At least in parts of Australia, there is a second powerful force militating against the production of quality materials. Many schools invest significant sums in purchasing instructional materials such as textbooks (and sometimes these days associated support materials), which are then loaned or hired to students. There are significant economic reasons for not changing text materials too often in such circumstances: school budgets will simply not permit this. As well as physical capital, many mathematics teachers have substantial intellectual and professional capital invested in the curriculum materials adopted by their school, so that too much is at stake to expect change to happen regularly. A consequence of tolerating very limited change over time is that at some point a more dramatic, and much more difficult, change is required to adapt curriculum to the resulting build up of new technological circumstances.

Anyone attempting to develop curriculum materials involving calculators will quickly confront the difficult question of how to deal with the fact that there are several different models and brands of calculators available. At one level, it seems unwise to embed key stroke commands into a text, unless there is some assurance that almost all readers, pupils and their teachers, are using the same calculator, which is of course increasingly unlikely. At a deeper level, there are differences in the functionality of calculators which are not always easy to accommodate. For example, to deal with solving a system of simultaneous linear equations, some calculator models require that coefficients be entered as a matrix, and that matrix operations be then performed. Others allow a matrix to be entered, but provide a solution immediately, if there is one, or an error if there isn't. Still others provide solutions using an RREF (row-reduced echelon form) command, which then requires an interpretation of the result. It is quite difficult to produce text material that sensibly accommodates to all of these at once. A partial solution is to provide supplements or appendices that tailor to particular graphics calculator models. Although this seems a bit clumsy, it may be the best available response in the short term. Developing curriculum materials is certainly not helped by regular changes to calculator capabilities, fuelled mainly by competition between manufacturers. Such changes provide further weight to the need to develop materials that are as device-independent as possible.

Curriculum materials do not always consist of pupil texts, of course. Another species consists of supplementary materials such as photo-reproducible masters for calculator activities or how-to books that provide detailed support for using a particular calculator, or suite of calculators, in an educational context. (Calculator manuals are rarely useful to beginners, unfortunately.) The latter may provide a temporary solution at an individual school with some control over which calculators are available to pupils, and may help teachers understand the possibilities and pitfalls of calculator use. However, the former run a grave risk of cementing in place a view of technology as an add-on, to be included at the behest of the teacher and the worksheet, rather than an integral part of the curriculum. By their nature, there seems to be a tendency, by no means universal, for supplemental materials of these kinds to focus on detailed key strokes, with the attendant risks that students might not be engaged in the desired kinds of mathematical thinking.

Assessment

As noted above, assessment is a key to understanding the attained curriculum. It is also a significant aspect of the implemented curriculum and of course an integral element of the official curriculum. This is especially so in the case of technology in general and graphics calculators in particular, since assessment structures that do not adequately incorporate technology will easily undermine any attempts to include technology in the curriculum. The evidence of the link between assessment regulations and calculator use in classrooms is overwhelming. In Australia, for example, graphics calculators are widely available and used by pupils in states which permit or mandate their use in formal assessment, and rarely used in those states which do not.

While a coherence between the everyday use of calculators and their availability in high-stakes assessment is clearly desirable, careful thought is required to bring this about effectively. Kissane, Kemp & Bradley (1996) outline and exemplify the range of possibilities in some detail. Essentially, assessment tasks need to be carefully designed to accommodate the use of technology, care needs to be taken to ensure that students are encouraged to learn mathematics, not just button-pushing, and equity issues should be borne in mind and minimised.

Recent changes to graphics calculators have thrust new challenges to assessment to the forefront again. Three of these are discussed in some detail by Kissane (2000): the availability of symbolic manipulation capabilities on algebraic calculators, the use of flash memory (which allows calculators to be upgraded and new capabilities added) and the expansion of available storage memory to a point at which very significant amounts of text information might be stored. One of the reasons that computers had limited impact on school mathematics curriculum is arguably that they have rarely been accommodated into assessment. Recent developments, reducing the now arbitrary (perhaps even merely linguistic?) gap between computers and calculators, are particularly interesting from this perspective. Research such as the University of Melbourne project (McCrae *et al*, 1999; Stacey *et al*, 2000) is especially important in this area.

Professional Development

Teachers are a crucial part of any process of curriculum change, and need to be supported in various ways. Through the lens of the implemented curriculum, the opinions and practices, beliefs and competencies of the teacher are a critical part of the curriculum. Curriculum developments that do not adequately provide for the legitimate needs of teachers do so at their peril.

It is vitally important to remember also the everyday working conditions of the great majority of mathematics teachers. In all countries, most teachers feel very considerable pressures of various kinds from their pupils, their school, administration, parents and the world at large. They frequently feel obliged to work within a curriculum context over which they have had little control, teaching more pupils than they would like, many of whom they feel should not be there at all. Many of their adolescent pupils have much more passionate interests than the mathematics classroom can satisfy, and indeed, derive more satisfaction from the social world of the school than its intellectual world. Increasingly, mathematics teachers are less well-qualified in mathematics itself. The pressures of external examinations, lesson preparation, motivating pupils, dealing with unruly pupil behaviour, marking and other responsibilities within the school all exact a toll on the proportion of the thinking and working time teachers have to adapt to new technologies such as graphics calculators. It is quite simply unreasonable to expect many teachers in these kinds of circumstances to shoulder the responsibility of adjusting the curriculum to the influence of graphics calculators without a good deal of help.

Morony & Stephens (2000, pp16-17) identify a number of dimensions of support in summarising the AAMT conference. These include the development of strategies to support practitioners who have reached different stages of comfort with graphics calculators, identified as 'novices', 'practitioners' and 'creators' in increasing order of sophistication. Good relationships among teachers both within and between schools need to be fostered and high quality resources for integrating calculators, going beyond the genre of worksheets and blackline masters need to be developed and disseminated.

Conclusions

In the past, many curriculum innovations have withered, and it is a common practice to talk of fads in education and to speak of pendulum swings of attention. It is not appropriate to approach technology in the mathematics curriculum from this perspective. Dangerous as it is to make predictions in the changing circumstances of technology, I suggest that we are not here dealing with another fad. Tucker (1999) expressed the new reality well:

> It pays to heed history. Technology always wins. The world may have been better when people walked instead of driving cars, but that is irrelevant. As long as there is gas, people will drive cars, and what I really care about is that they drive them sensibly. The mathematical world may have been better when people did arithmetic or graphed functions on paper or in their head instead of on a calculator, but that is irrelevant. As long as there are batteries, students will use calculators, and what I really care about is that they use them sensibly. … pretending something doesn't exist is not a good teaching strategy. For many of my students, graphing calculators are as much a part of their intellectual constitution as pencil and paper, and I have to learn to deal with it. (p.910)

As many, such as Penglase & Arnold (1996), have suggested, the significance of the graphics calculator derives in the first place from its affordability and thus its accessibility. The world external to education has brought this situation about, and, in the near term, can be expected to continue it. We need to respond to that reality as soon and as well as we can, guided by sound research and good practice.

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