THE IMPORTANCE OF BEING ACCESSIBLE: THE GRAPHICS CALCULATOR IN MATHEMATICS EDUCATION*

Barry Kissane
Murdoch University
Murdoch, Western Australia, 6150
kissane@murdoch.edu.au

The first decade of the availability of graphics calculators in secondary schools has just concluded, although evidence for this is easier to find in some countries and schools than in others, since there are gross socio-economic differences in both cases. It is now almost the end of the second decade since the invention of microcomputers and their appearance in mathematics educational settings. Most of the interest in technology for mathematics education has been concerned with microcomputers. But there has been a steady increase in interest in graphics calculators by students, teachers, curriculum developers and examination authorities, in growing recognition that *accessibility* of technology at the level of the individual student is the key factor in responding appropriately to technological change; the experience of the last decade suggests very strongly that mathematics teachers are well advised to pay more attention to graphics calculators than to microcomputers.

There are clear signs that the commercial marketplace, especially in the United States, is acutely aware of this trend. It was recently reported that current US sales of graphics calculators are around six million units per year, and rising. There are now four major corporations developing products aimed directly at the high school market, with all four producing graphics calculators of high quality and beginning to understand the educational needs of students and their teachers. To get some evidence of this interest, I scanned a recent issue (April 1995) of *The Mathematics Teacher*, the NCTM journal focussed on high school mathematics. The evidence was very strong: of almost 20 full pages devoted to paid advertising, nine featured graphics calculators, while only two featured computer products, with two more featuring both computers and graphics calculators.

The main purposes of this paper are to explain and justify this heightened level of interest in graphics calculators at the secondary school level, and to identify some of the resulting implications for mathematics education, both generally, and in the South-East Asian region.

DESCRIPTIONS AND DEFINITIONS

In some respects, graphics calculators are similar to other calculators, such as scientific calculators and four-function calculators, which have become quite familiar to mathematics teachers over the last two decades. The most notable similarities are that each is small enough to be hand-held, has an independent power source, is operated by pressing keys, has a monochromatic numerical display and is quiet in operation. Because of these surface similarities, it is not surprising then that the term 'calculator' was used by manufacturers to describe this new form of technology.

However, the differences between graphics calculators and other kinds of calculators are much greater than the similarities, and has given rise to substantial misgivings about the use of the term 'calculator' to describe such different devices. Perhaps the most obvious difference is the graphics display screen. As the name suggests, a graphics calculator screen can be used to display graphs of functions or of statistical data, both clearly of considerable value in mathematical work. This property, together with typical advertising images, has given some mathematics teachers the impression that a graph-drawing capability is the distinguishing feature of a graphics calculator.

This is not the case, however. From the perspective of mathematics education, the most important difference between a graphics calculator and its ancestors is not the display screen, but the mathematical capabilities built into the device. (As an aside, it is interesting that the developers of the first 'supercalculator', Hewlett Packard's HP-28, added the graphing capabilities as an afterthought to the rest of the mathematical software embedded in the calculator, and certainly did not think of them as the major innovation of the device.) Although there are many differences among them, most currently available models contain a significant suite of mathematical capabilities, including the standard functions found on scientific calculators, together with function graphing (on rectangular or polar coordinates, with explicitly, parametrically or recursively defined functions), manipulation of graphs, numerical equation solving and root-finding, data analysis (both numerically and graphically), matrix manipulation, operations with sequences and series, complex number arithmetic and numerical differentiation and integration. In addition, all are programmable, and have considerable memory for longer-term storage of programs, data, matrices and images. (Indeed, several calculators on today's market have more user-accessible memory than did microcomputers of the late 1970's.) Some calculators have limited symbolic manipulation capabilities as well. Modern graphics calculators have some communication capabilities, to other calculators, computers, printers or overhead projection units.

A major consequence of these kinds of capabilities is that a graphics calculator can be used to *analyse* a mathematical situation rather than merely to perform a computation, an enormous surge of mathematical power, when compared with the scientific calculator. The scientific calculator actually provided students with little more than the four-function calculator. The most obvious advances were table facilities (replacing the previous need to have trigonometric, logarithmic and exponential tables), statistical facilities (which only replaced arithmetical aspects, since actual data were not stored for analysis) and programming facilities (which in fact were rarely used in schools). One could be forgiven for thinking that the graphics calculator is, like the scientific calculator before it, merely a slight advancement in terms of mathematical power. But one would be wrong.

Alternative names for graphics calculators have been suggested. Two in everyday use are 'graphing calculator' and 'graphical calculator'. There are also compound names, such as 'programmable graphical calculator', 'graphing scientific calculator' or 'advanced graphic calculator', but it is not clear whether these are attempts to grapple with the conceptual problem of adequately defining the devices or whether they are motivated by marketing issues. More expressive suggestions have been made, including 'supercalculator' and 'compulator', each of which acknowledges a growing unease with the term, 'calculator'. It is worth noting the historical precedent for difficulties with describing devices of these kinds. There is not much 'scientific' about scientific calculators and four-function calculators almost always have more than four functions.

Whatever it is called, however, it is preferable to think of the graphics calculator as a small, portable computer with inbuilt mathematical software that costs substantially less than other kinds of computers. It is arguably the most potent influence on the secondary mathematics curriculum of today, and particularly on the curriculum of tomorrow.

GRAPHICS CALCULATORS OR COMPUTERS?

Until recently, technology in mathematics education referred mainly to computing technology, fuelled by the extraordinary rise to prominence and reduction in price of the microcomputer over the past two decades. In many respects, such 'high' technology continues to excite both mathematics educators and mathematicians alike. This is especially so in the past decade when a number of significant developments in software for mathematics and mathematics education have been developed. Some examples of such software are *Derive*, *Mathematica*, *Theorist*, *Maple*, *Logo*, *Cabri-géomètre*, *Geometer's Sketchpad*, *AutoGraph*, *MousePlotter*, *ANUGraph* and *MiniTab*. In affluent western countries, many secondary schools have acquired significant microcomputer resources, usually in the form of laboratories full of machines. A small number of elite schools have even begun to equip each of their students with portable laptop computers and to plan their curriculum accordingly. Schools with less resources have opted for a configuration of a demonstration computer in each classroom, or a mobile computer that can be shared between classrooms.

While such developments at the leading edge of high technology are exciting and challenging, the reality for many students in more affluent countries, and for almost all students in less affluent countries is that microcomputers are too expensive and hence not a significant force on mathematics education. In short, the technology has been, and continues to be, inaccessible to the great bulk of students for almost all of their time in school. Further, since agencies responsible for official curricula naturally attend to the circumstances surrounding the mass of students, rather than an elite few, there have been almost no significant effects of microcomputers on the mathematics curriculum prior to undergraduate education.

As the title of this paper suggests, the key to understanding the significance of graphics calculators is their potential for increasing the accessibility of technology to individual students. There are two aspects to this accessibility. In the first place, the purchase price of graphics calculators, while still too high for many individual students, places them within reach of many more classrooms than do microcomputers. Schools can purchase a class set of graphics calculators for around the same price as a single microcomputer sufficiently powerful to operate modern innovative software. This is certainly the case if the cost of the software is taken into account, since graphics calculators come complete with their own mathematical software, while computers demand that the software be purchased separately. Even for individual students, the cost in present terms of a graphics calculator, spread over several years of schooling has become comparable with the cost of scientific calculators late in the 1970's, especially if students do not need to purchase a scientific calculator as well. The remarkable surge in recent sales of graphics calculators in affluent western countries suggests that many schools and individuals find them affordable.

The second aspect of accessibility is a consequence of the physical size of graphics calculators. Small, light, battery-operated computers are clearly much more portable than are large, heavy, electrically-powered computers. Graphics calculators are as potentially mobile as the students for whom they were designed. They can easily be taken home and they can accompany students to an examination room or on a field trip. They can easily be moved around a school; in my institution, we have a briefcase containing a set of graphics calculators, which allows our 'computer laboratory' to be wherever we and the students are, rather than the *much* more difficult problem of transporting the students to a computer laboratory. (Bradley, Kemp & Kissane, 1994) As long ago as 1986, the University of Chicago School Mathematics Project, developing an innovative 11th grade course (Rubenstein et al., 1992) based on a premise of significant computer access, found that schools were much more likely to be able to acquire access to graphics calculators than to computers, which were often used by computing subjects, were located inconveniently in computer laboratories and required too much advance booking of rooms to be a realistic option.

Despite their relative inaccessibility and price disadvantage, it should be acknowledged that microcomputers enjoy some significant advantages over graphics calculators for secondary mathematics education. They are much more powerful, are faster and can use much more mathematically and educationally sophisticated software. They are much more versatile, in the sense that the same computer can be used for many different purposes. Computer screens are larger and have higher resolution than current graphics calculator screens, and are generally coloured while (most) graphics calculator screens are monochromatic; thus more information can be presented more effectively. Computer software is more easily upgraded and modified than is graphics calculator software. Computers rely on electricity rather than batteries, which are a nuisance to replace. (Although this is not always an advantage for computers; I heard recently of one South-East Asian country in which many schools were issued computers from a central government, although they lacked adequate electricity supplies to operate them.) It is usually easier to print from a computer than a graphics calculator. However, these many advantages of microcomputers over graphics calculators evaporate and are merely of academic interest if students do not enjoy ready access to machines.

While it would be incorrect to claim that there is not a high technology element in today's graphics calculators, and no more correct to regard them as 'low' technology, it seems more reasonable to regard them as an example of what Schumacher referred to as '... *intermediate technology* to signify that it is vastly superior to the primitive technology of bygone ages but at the same time much simpler, cheaper and freer than the super-technology of the rich.' (1974, p.128) Graphics calculators are *much* more appropriate than are microcomputers to the realities and constraints of most students in most classrooms in most countries at this moment in time, and for at least the next few years, and are thus a form of intermediate technology for school mathematics education.

METAPHORS FOR GRAPHICS CALCULATORS

The previous section indicated flaws in using the metaphor of a calculator rather than that of a computer to think about graphics calculators. In fact, it seems that people invoke a number of metaphors to help them come to terms with graphics calculators and other computers relevant to mathematics education. These are described in detail in Kissane (1995c) and include the following:

Laboratory. The calculator provides opportunities for exploration of mathematical ideas and situations, akin to the explorations characteristic of scientists in a laboratory. Both doing and learning to do mathematics have significant elements of personal exploration associated with them, and the calculator provides a powerful environment for such things to occur.

Tool. The calculator provides a tool for doing a particular mathematical task, so that learning when to use it, when not to use it and how to use it well are as important for students learning mathematics as the equivalent learning is for people learning the tools of other trades.

Teaching aid. Since computers are often regarded as teaching aids (in part because of their limited availability), it is not surprising that some people think of graphics calculators as devices to help teachers to teach more than as devices to help students to learn. The availability of overhead projection capabilities strengthens this metaphor.

Curriculum influence. The accessibility and portability of the calculator demand that serious attention be paid to whether we are teaching the right things, in the right way at the right time to the right people. It is especially significant that it is the *central* elements of the secondary curriculum that are most susceptible to the influence of the graphics calculator, notably the traditional algebra, trigonometry, calculus sequence as well as statistics and probability, as elaborated below.

Status symbol. As for other areas of technology, it is inevitable that some will focus attention on next year's models or the features that one calculator lacks in comparison with others. Marketing people are naturally sensitive to this orientation, which is certainly not restricted to graphics calculators, but is quite evident with other computers too.

Cheating device. There is a strong tradition within mathematics education communities for doing mathematics the 'right way', and hardly surprising that some people's orientation to new ways of doing things is to regard them as illicit. The continuing strong influence of formal examinations in mathematics is also a factor here.

In recent work with first year undergraduate students, (Kissane, Kemp & Bradley, 1995) found evidence of each of these underlying metaphors for thinking about graphics calculators, and also suggested that an additional metaphor of the graphics calculator as a *nuisance* seemed to characterise the reactions of some students, who regarded the intrusion of graphics calculators into a course as merely adding to the burden of what is to be learned in an already crowded curriculum.

The significance of these various metaphors is that they help us to understand why there is a range of reactions from both students and their teachers to the use of graphics calculators, along a spectrum from unbridled optimism to hostility and derision. It seems important for *each* of these metaphors to be brought to the forefront in discussions about technology in general, and graphics calculators in particular, in order to achieve a balanced appraisal of each of the prospects, pitfalls and possibilities associated with technological change.

CURRICULUM IMPLICATIONS

There are considerable implications for the curriculum of the widespread availability and use of graphics calculators, as Romberg (1992) recently observed:

> Computers and calculators have changed the world of mathematics profoundly. They have affected not only what mathematics is important but also how mathematics is done. It is now possible to execute almost all of the mathematical techniques taught from kindergarten through the first 2 years of college on hand-held calculators. This fact alone must have significant effects on the mathematics curriculum. ... the changes in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics. (p.772)

There are implications for what is taught in secondary school mathematics, for how it is taught and for when it is taught. There is not space in this paper to do justice to the complete range of these implications; rather a selection is made.

The algebra curriculum has long been the central core of high school mathematics, and has concentrated upon symbolic manipulation to deal with both expressions and equations. Development of manipulative skill in algebra has been a major goal, based on the premise that no further progress in mathematics is possible without a fluent grasp of these skills. Evidence for success of this approach has been generally disappointing, with many students apparently acquiring the skills at the expense of the associated understanding of algebraic concepts and even the whole notion of generalisation. Until quite recently, there was limited emphasis on graphs and graphing in school algebra for the practical reason that it takes students so long to draw graphs that there is little time left to make use of them.

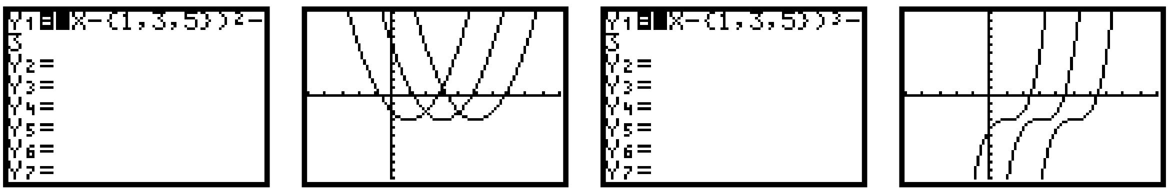

Figure 1: Calculator screens showing transformations of standard functions

The graphics calculator has allowed much more emphasis to be placed on graphs and their interpretation, both to help students understand key ideas (such as those of function, gradient and transformation) and to deal with practical algebraic problems (such as solving equations). The ease with which calculators can draw graphs means that students can concentrate on the meanings inherent in graphs instead of the mechanics of producing them. That is, the important curriculum task becomes to *make use of* a graph, rather than to *produce* a graph, which is a profound shift. Figure 1 (and other calculator screens in this paper) shows some calculator screens from a Texas Instruments TI-82™ graphics calculator. These screens illustrate how students might easily graph families of functions to help them understand horizontal transformations by studying many examples.

Modern graphics calculators provide a capability not unlike that of some innovative computer software to provide users with ready access to three different representations of functions simultaneously. These three aspects, sometimes referred to as 'the rule of three' are the symbolic, graphical and numerical respectively represented on calculators by symbols, graphs and tables of values. Figure 2 shows an example of this. There are clear advantages to understanding the nature of generalisations associated with seeing their particular manifestations in graphical or tabular (numerical) form, and also seeing the effects of changing the symbolic representation on the graphical and the numerical representations. In addition, the availability of graphics calculators suggests a better use of classroom time for exploration and conceptual development as well as applications of algebra to real situations, rather than an exclusive focus on the development of algebraic skills.

Another key idea in school algebra is that of equations. The graphics calculator has considerable impact on ways of dealing with this. Both graphical and the numerical representations of situations give rise to efficient and insightful ways of seeking solutions to equations. In addition, modern graphics calculators have an automatic solve command, so that numerical solutions to elementary equations are provided. An extended description of the range of possibilities is given in Kissane (1995b). Prior to the availability of technology such as graphics calculators, algebra curricula were mainly constrained to the solution of linear equations or equations for which factorising was appropriate. Iterative and approximate solution strategies were not technically feasible, and so were neglected. Now, there are serious doubts on whether factorisation is worth the trouble and time that it takes to teach and learn, at least if high levels of skill are expected. There has long been a perfectly good alternative to factorising quadratic expressions in order to solve quadratic equations – the quadratic formula – but this does not seem to have diminished enthusiasm for teaching young students about factorising trinomials. Only time will tell whether other sacred cows of the mathematics curriculum, such as exact values of trigonometric functions and a preoccupation with trigonometric identities will endure. However, the curriculum continues to be a zero-sum game, so that if new techniques and ideas are to be included, something must be removed to make room.

At present, symbolic manipulation capabilities of graphics calculators are rather limited, where they exist at all, but this is a temporary state. Recent developments such as the Texas Instruments TI-92™ and the Hewlett Packard HP-38G™ calculators have inbuilt symbolic manipulation capabilities and it seems reasonable to expect this trend to increase. Once again, although the capabilities are likely to be much more limited than those of fully fledged computer algebra systems, the calculator accessibility advantage is critical. Opinions will be divided for a while on the merits of allowing students access to automatic symbolic manipulation For example, Waits & Demana (1992) suggested that graphics calculators provided more support for algebraic intuition than did symbolic manipulation software at that stage while French (1993) anticipated the forthcoming debate:

> The implications for the teaching and learning of algebra are immense, because so much of the traditional development of manipulative skills must be called into question. If simplification, factorisation and equation solution are available at the press of a key, as well as the means of plotting graphs, we are again faced by questions about the algebraic understanding and skills that children need to develop and the opportunities, as well as the difficulties, created by such a powerful tool. (p. 18)

Some recent curriculum development projects concerned with algebra have proceeded on an assumption of access to graphics calculator technology, including senior school courses such as the Nuffield Advanced Mathematics Project (Nuffield Foundation, 1994) in the UK and the University of Chicago School Mathematics Project (Rubenstein et al., 1992) in the USA as well as lower secondary school courses, such as *Access to algebra* (e.g., (Lowe et al. 1994; Lowe et al., 1994) in Australia. Indeed, Heid (1995) suggests that the availability of technology to students will affect profoundly the algebra curriculum across the whole range of schooling. A clear shift in emphasis away from symbolic manipulation for its own sake and towards the use of algebra to understand and model real situations is evident in all of these curriculum development initiatives.

Although it is still too early to judge attempts to use graphics calculators in these ways, the early signs are encouraging. In a recent review of research, Dunham & Dick (1994) concluded:

> The early reports from research indicate that graphing calculators have the potential dramatically to affect teaching and learning mathematics, particularly in the fundamental areas of functions and graphs. Graphing calculators can empower students to be better problem solvers. Graphing calculators can facilitate changes in students' and teachers' classroom roles, resulting in more interactive and exploratory learning environments. (p. 444)

It is still rather difficult to get good research evidence on the desirability of various kinds of curriculum change in algebra, however, since the contexts of the research are still often fairly artificial. At present, it is still necessary to make inferences about the impact of graphics calculators on the learning of students in classrooms organised to take advantage of the technology, over substantial periods of time and following curricula that have been designed to incorporate it appropriately, since direct evidence is not yet available.

As for algebra, there are considerable implications for elementary calculus of the availability of graphics calculators. Like algebra, calculus in high school and the early undergraduate years has long been learned (although not necessarily taught) as a collection of symbolic manipulative skills, with too little time left to focus on the important concepts involved. This has not gone unnoticed, of course. As one senior Australian mathematician recently put it, ‘Students come into my university with well-developed skills at integration, but with almost no idea of what integration is *for*.’ Access to graphics calculators has the potential to concentrate student attention on the big ideas of the calculus, and attend to the development of appropriate manipulative skills later.

An example of this is the notion of a derivative. Standard calculus courses deal with the slope of a tangent to a curve as the limiting case of secants to the curve through a particular point. Derivations from first principles are generally unconvincing to students, who focus instead on their symbolic consequences, consisting of various rules for finding derivative functions. With personal access to graphics calculators, it is possible for students to explore the gradient of a curve *directly* by successively zooming in on a curve. Functions that have derivatives are ‘locally straight’ and so it is technically unnecessary to deal with the tangent at all. Modern graphics calculators allow students to trace a curve, numerically evaluating the derivative at any point, leading naturally to the idea of a derivative *function*, which can be automatically graphed, as shown in Figure 2.

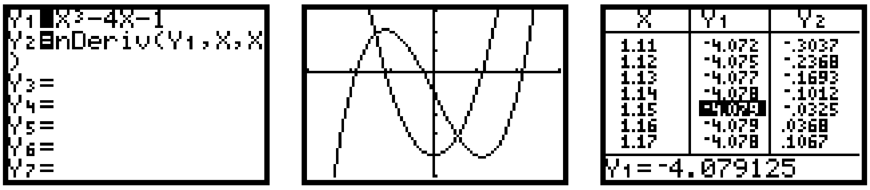

Figure 2: Three representations of a function and its derivative

In the past, many students seemed to regard derivatives as *expressions* rather than as functions; the conceptual value of graphing a function and its derivative simultaneously has only recently been appreciated. In Figure 2, the turning points of function Y_1 are just the points where its derivative crosses the *x*-axis. A tabular representation of the same information is also shown, giving numerical values for the function and its derivative at points near the relative minimum point. Students can toggle between these three representations to help them make connections between them, and can readily see the effects of changing the original function on the graphs or table. With such representations at their fingertips, students have much richer access to the important ideas of the calculus than are afforded by our traditional preoccupation with symbolic manipulation.

A major endpoint of studying the derivative in elementary calculus is to find relative extrema of functions. Modern graphics calculator allow these to be found numerically and directly, long before calculus is studied. For example, the topic of optimisation appears naturally in Lowe et al. (1994, p.103), where it is treated numerically, graphically and informally, typically a full year before it would appear in a calculus course. In addition, most graphics calculators have various commands for finding extreme values numerically, such as the two illustrated in Figure 3.

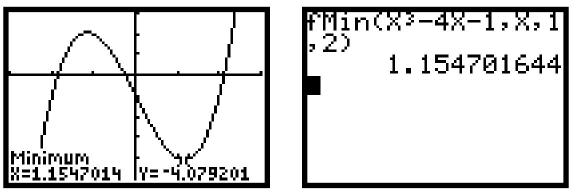

Figure 3: Automatic determination of a relative minimum

Numerical procedures do not provide exact answers, of course, which explains why the two results shown in Figure 3 are accurate to only five places of decimals. However, such a level of accuracy would meet any genuinely practical purpose plausibe to students. The numerical procedures also provide students with a good intuitive grounding and some motivation for later study of exact procedures through a symbolic calculus

There are a number of other ways in which the calculus curriculum might be affected (generally for the better) by student access to graphics calculators. Some details are given in Kissane (1995a). The calculus reform movement in the USA has been substantially influenced by the development of graphics calculator technology. One of the people involved in this movement (Kennedy, 1994) summed up the sentiments of many fellow reformers recently:

> We have been teaching the math literate of tomorrow with the problems of yesterday, while explaining to them that they will *need this mathematics in the future*. My friends, this is the stuff of which the Emperor's New Clothes are made! While we have been spinning golden oldies on the phonograph, perhaps more accurately the victrola, the world or Mathematical Reality has gone CD." (p.607)

Finally, a third example of the implications of graphics calculators for school mathematics concerns data analysis. A scientific calculator allows students to derive some numerical statistics (such as means, standard deviations and regression coefficients) from a set of data. In contrast, and like other computer packages, a graphics calculator actually *stores* the data so that transcription and entering errors can be detected, data can be sorted, alternative analyses can be performed, graphs can be produced, relationships examined using scatterplots, outliers removed and the data generally *analysed*, rather than merely summarised. Hackett & Kissane (1993) explore the implications of these sorts of capabilities in more detail. Figure 4 shows some examples of the data analytic capabilities of the TI-82™, showing some bivariate data (with one set sorted), a histogram and statistics for the first variable, and a pair of box plots allowing the two distributions to be compared.

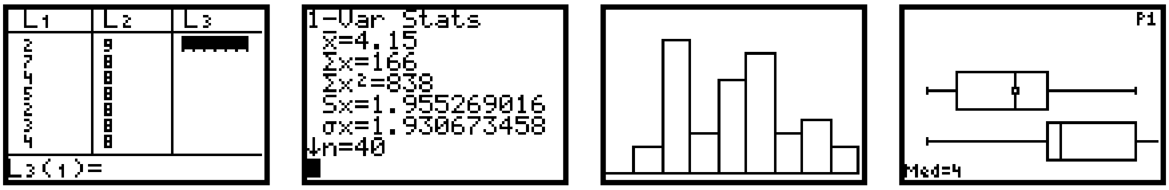

Figure 4: Some data representations from a graphics calculator

These four screens illustrate the general point that a modern graphics calculator allows students to engage in the same kinds of exploratory data analysis as they would do on a microcomputer; once again, the relative accessibility of the graphics calculator gives it a significant advantage.

This section of the paper is already long, yet contains only a few examples of possible influences of graphics calculators on the mathematics curriculum . It is important to bear in mind that this development does not impact only on aspects of the curriculum of marginal and passing interest; many aspects of the traditional *core* of the secondary curriculum can be taught and learned differently, and may be subject to a new kind of curriculum scrutiny in the light of this technology, as the earlier quote from Romberg suggests. This kind of scrutiny has not been undertaken with microcomputers in mind, since there have rarely been enough physical facilities available for the results to justify the considerable effort needed to obtain them. But that has now changed. If it becomes possible that most, or even very many, students can have personal access to graphics calculators, then a serious examination of the curriculum implications will be worth undertaking.

ASSESSMENT ISSUES

Successful incorporation of technology into mathematics curricula is only possible if careful thought is given to issues of assessment. The critical idea is that there should be a coherence between the conditions in which students normally learn and do mathematics and those in which their achievements are formally assessed. In framing their US mathematics curriculum standards, the National Council of Teachers of Mathematics (1989) was unambiguous in its recommendation regarding technology, suggesting that all high school students should have access to a scientific calculator with graphing capabilities at all times. Without a high level of integrity of this kind, there is little prospect of the technology being regarded as important by teachers or students alike. Indeed, the sorry history of the microcomputer in mathematics education is readily explained with reference to this precise point, since it has been too hard to provide students with enough computer access to realistically assess their mathematical achievements. The portability of graphics calculators resolves much of this problem, but at the same time exposes other problems to be addressed.

Assessment can take many forms in mathematics, ranging on a continuum from casual informal observation of students in their classrooms, through submitted work such as assignments and projects to more formal means of assessment such as classroom tests and timed external examinations. There would seem to be few problems associated with integrating graphics calculators into less formal assessment processes; the major issues are related to the more formal settings of tests and examinations. The three most pressing issues concern the styles of questions appropriate to graphics calculator use, the possibility of some students having an advantage over other students because of the use of different models of graphics calculator with different features and the emerging problems associated with symbolic manipulation capabilities of calculators. These issues are much less problematic at a local level (say, an individual classroom or school) than they are at a regional (state) or national level. Of course, they are not problems at all for settings in which mathematics examinations are not relied upon for assessment, but there are still very few of these.

Experience related to the first issue is accumulating, since UK Examination Boards have permitted the use of graphics calculators in A-level examinations for some years now, and there have been isolated experiences of other kinds reported as well (E.g., Kissane, Bradley & Kemp, 1994). Significantly, the US College Board has allowed graphics calculator use for the Advanced Placement Calculus AB and BC examinations since May 1995. The new arrangements include some questions (both multiple-choice and free response) for which calculators are *required* and others for which students are not permitted to use a graphics calculator. In addition, rather than demanding that students clear their calculator memories before the examination begins (a practice apparently used in the UK examinations, although it is hard to imagine how it can be successfully invigilated), students are encouraged to use the programming features of calculators to ensure that their calculator has a minimum set of capabilities. The College Board itself provides suitable programs for students to enter into their calculators for this purpose, and suggests that these be well understood before the examinations begin. In this way, the examiners can be reassured that each student has available to them a facility for graphing, numerical equation solution, numerical integration and numerical differentiation. Although not a complete solution, this seems to be a good way of reducing some of the perceived inequities among students.

As far as question styles are concerned, there is a considerable difference between *requiring* and *allowing* a graphics calculator to be used. In the former case, questions are asked which cannot be reasonably be answered by students without using a graphics calculator, and it is expected that part of the assessment task is to decide when calculator use is a good idea and when it isn't. When calculators are merely allowed on examinations, there is a tendency to try to ask calculator-neutral questions. Bradley (1995) has noted that one common way of doing this is to replace some numbers with algebraic symbols, which unfortunately may have the effect of making questions harder than intended. In addition, setting examination questions to avoid any advantage being conferred on graphics calculator use is not consistent with sound integration of technology into the curriculum referred to above, and may ultimately be counter-productive. Presumably, the reason for 'allowing' rather than 'requiring' graphics calculator use is that some students do not have graphics calculators yet, or have not yet had sufficient time to become fluent with them. If this is a temporary problem, its solution will be delayed by avoiding the issue of what is important when *everyone* has a graphics calculator available to them. One of the few advantages of timed external examinations is the prospect that they might be used as a fairly powerful form of encouragement to schools to change in some ways. However, the use of calculator neutral papers would seem to undermine this potential. Once again, solving one educational problem seems to create others.

Care is needed to deal with what students actually write down, since graphics calculator use can often permit students to provide a numerical answer only; if an explanation is to be given, it seems important that students be explicitly advised about this. Similarly, students need to be informed when exact answers are expected and when a numerical approximation is adequate, and when they should make (and defend) a choice between these. These and other issues of these kinds are also referred to by Kemp & Kissane (1995) and Kissane et al. (1994). The little experience so far formally gathered and published on the use of graphics calculators in examinations suggests that students do not generally make best use of the potential of the devices, according to Bradley (1995)

The issues associated with symbolic manipulation are more difficult to resolve, as noted by Bradley (1995), but are already in need of urgent attention with the latest batch of new models of graphics calculators including significant symbolic capabilities. The 'solutions' of prohibiting the calculators for examination use or of disabling the symbolic capabilities may provide a temporary respite from the problem, but certainly are not sufficient for the longer term.

A particular problem in many places is the speed of change; technological change tends to be frighteningly fast while educational change tends to be extraordinarily slow. Thus, in most Australian states, schools are given at least two years advance notice of significant changes to examinations; in the UK, examination papers tend to be set a full two years in advance of their administration. For many elements of society, even two years notice of impending change is inadequate. School systems, publishing companies and many teachers, students and their parents have great difficulty coming to terms with significant change. On the other hand, graphics calculators are prone to change enormously over the course of two years, especially now that there are four significant international corporations competing for the same market.

Predicting the future is hazardous at the best of times; predicting the future developments in graphics calculators seems an especially error-prone activity, and not for the faint-hearted. It is clear that the development of symbolic algebraic capabilities on inexpensive graphics calculators raises difficult problems for assessment. The same is likely to be true for the development of dynamic geometry systems, spreadsheet capabilities and improved screen resolution, as well as other enhancements of graphics calculators. The next few years are likely to be difficult ones for people responsible for mathematics examinations, trying to chart a course between changing too slowly and changing too rapidly, when it is not really clear which is the greater evil.

CONDITIONS FOR IMPLEMENTATION

There are remarkably few voices in the professional literature opposed to the integration of technology into mathematics education, and even today's suite of available graphics calculators have been well enough designed to be enormously useful to students learning mathematics and teachers teaching them. The main barriers to the more widespread educational exploitation of this technology are human and financial rather than technical. To describe the conditions for successful implementation in detail would require a companion paper to this one, but this section will summarise the main requirements evident at the moment.

The most critical factor is likely to be the financial one. Graphics calculators remain relatively expensive (compared with paper and pencil, chalk, textbooks and other bare necessities of mathematics education) even though they are relatively inexpensive compared with microcomputers and their software. Although there is now a relatively inexpensive graphics calculator available from one manufacturer (the TI-80™ from Texas Instruments), most of the development seems to be focussed on producing calculators with improved features. As for microcomputers, prices have stabilised and the products are improving. For many schools and their students, the stabilised price is too high, however, especially in developing countries. At the very least, strategies of directing educational resources towards graphics calculators instead of computers would seem to be wise, but a companion strategy of producing inexpensive calculators with basic features may also be a good idea. There is scope here for productive partnerships between education and industry.

The provision of graphics calculators hardware is a necessary but not a sufficient condition for effective change, however. The frequently forgotten element in curriculum development, the classroom teacher, is crucial. She will need help, support and encouragement to use graphics calculators well. Professional development support in the form of courses, time to attend them, and associated materials are all needed. (E.g., Andrews & Kissane, 1994) Teachers need to have a personal graphics calculator and some time to become familiar with it before they will be confident to use one in their classrooms; there seems to be no effective substitute for this personal experience. Curriculum development to take better account of the potential of graphics calculators is also needed, some examples of which are hinted at earlier in this paper. It is unwise to leave all use of the graphics calculator until the final year or two of secondary schooling. Suitable student text materials, advice for teaching in an environment that includes ready access to graphics calculators, and an assessment environment that accommodates graphics calculators well are also crucial.

This is a formidable list, which helps to explain why educational change is so hard to bring about. But there is scope to learn from each other about many aspects of curriculum development, and a redirection of some energies and resources away from microcomputers towards the more appropriate technology of graphics calculators may help to bring about change. We are well reminded, in trying to bring about change of a recent suggestion by Leinwand (1994):

> It is unreasonable to ask a professional to change much more than 10 percent a year, but it is unprofessional to change by much less than 10 percent a year. (p. 393)

CONCLUSION

Graphics calculators are small portable computers with built-in mathematical software. It is restricting and unwise to think of them as calculators. Their potential for school mathematics education is not restricted to their graphic capabilities. While they make good mathematical tools for students, there are other appropriate metaphors for them. Accessibility of technology is critical, but means more than accessibility to the physical device. Educational environments designed and nurtured to take good educational advantage of the potential of graphics calculators in curriculum, teaching and assessment are needed. The issue of accessibility to technology suggests a need for more thoughtful allocation of resources for school mathematics. Graphics calculators represent a more appropriate technology for mathematics education than do microcomputers. While this seems to be the case generally, it seems especially so for less affluent nations, including developing nations in the South-East Asian region. Finally, to gain maximum educational benefit from this form of intermediate technology, careful attention to the needs of classroom teachers of mathematics is required.

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